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Holographic dual of the standard model on the throat

Juan F. G. Cascales,[a] **Fouad Saad**[a] **and Angel M. Uranga**[ab]

[a] *Instituto de Física Teórica, Facultad de Ciencias*
Universidad Autónoma de Madrid, 28049 Madrid, Spain
[b] *Theory Division, CERN*
CH-1211 Geneva 23, Switzerland
E-mail: juan.garcia@uam.es, fouad.saad@uam.es, angel.uranga@uam.es

Abstract: We apply recent techniques to construct geometries, based on local Calabi-Yau manifolds, leading to warped throats with 3-form fluxes in string theory, with interesting structure at their bottom. We provide their holographic dual description in terms of RG flows for gauge theories with almost conformal duality cascades and infrared confinement. We describe a model of a throat with D-branes at its bottom, realizing a 3-family Standard Model like chiral sector. We provide the explicit holographic dual gauge theory RG flow, and describe the appearance of the SM degrees of freedom after confinement. As a second application, we describe throats within throats, namely warped throats with discontinuous warp factor in different regions of the radial coordinate, and discuss possible model building applications.

Keywords: Gauge-gravity correspondence, Superstring Vacua, Brane Dynamics in Gauge Theories, Flux compactifications.

Contents

1. Introduction

Theories with strongly warped extra dimensions (warped throats) have revealed novel features compared with standard factorised compactifications [1, 2]. Such theories have been intensively applied to phenomenological model building beyond the Standard Model (SM) to address a variety of questions. The prototypical example of such applications is the RS1 construction [1], where a slice of AdS_5, with two boundaries, is regarded as a 5d compactification to 4d with an exponential warp factor in the extra dimension. Location of SM fields at the strongly warped end (infrared brane) leads to an exponential suppression of 4d scales as compared with the mildly warped end (ultraviolet brane), thus providing a new approach to the Planck/electroweak hierarchy. Inspired by the AdS/CFT correspondence in string theory, results in RS phenomenology have been interpreted in purely 4d terms by replacing the warped throat by a strongly interacting 4d conformal field theory. However, lack of a microscopic understanding of holography in the effective field theory approach prevents this picture to go beyond a qualitative rephrasing.

A microscopic construction of warped throats and their holographic description can be obtained in string theory. Warped extra dimensions appear in string theory in compactifications with non-trivial field strength fluxes, due to their backreaction on the underlying metric [3, 4]. In particular, warped throats with exponential warp factors appear when fluxes are (in intuitive terms) associated to 3-cycles localized in small regions of the compactification space.

The prototypical example is provided by the Klebanov-Strassler throat [5]. It is based on the local geometry of the deformed conifold. One turns on M units of RR 3-form flux on the $\mathbb{S}^3$ at its tip, and a suitable density of NSNS 3-form flux on its dual non-compact 3-cycle. If the latter is considered compact by setting a cutoff distance (or by embedding in a global compactification), one denotes by K the total NSNS flux. For $K \gg g_s M$, where g_s is the IIB string coupling, fluxes backreact on the geometry and create a warped throat, whose geometry is approximately $AdS_5 \times T^{1,1}$ (with the cosmological constant of AdS_5 and the 5-form over $T^{1,1}$ slowly varying along the radial direction). The roles of the IR and UV branes in RS1 are played by the $\mathbb{S}^3$ of the deformed conifold and the cutoff/compactification, respectively. The relative warp factors between both endpoints is $e^{-\frac{2\pi K}{3g_s M}}$ [6, 5, 4]. The holographic dual description is given by the (almost conformal) $\mathcal{N}=1$ supersymmetric 4d gauge theory on $N = KM$ D3-branes at the singular conifold in the presence of M fractional branes. The gauge theory [7] has a gauge group $\mathrm{SU}(N) \times \mathrm{SU}(N+M)$, chiral multiplets A_i, B_i, $i=1,2$ in the representations $(\Box,\overline{\Box})$, $(\overline{\Box},\Box)$ respectively, and a superpotential $W = \epsilon^{ij}\epsilon^{kl}\,\mathrm{tr}\, A_i B_k A_j B_l$, which is marginal for the strongly interacting conformal theory. Along the renormalization group (RG) flow to the infrared, the theory suffers a cascade of Seiberg dualities [8], in which the effective N decreases in steps of M. Eventually, at an infrared scale achieved after $K = N/M$ duality steps, the theory confines and running stops. This corresponds to the IR ending of the dual throat, with the infrared confinement scale related to the $\mathbb{S}^3$ size. The warp factor is associated to the ratio of UV and IR scales $e^{-\frac{2\pi K}{3g_s M}}$ generated by the RG flow.

Unfortunately, the Klebanov-Strassler throat is too simple to e.g. generate chiral physics in the infrared. On the gravity side, the geometry is smooth after the deformation, while in the field theory side, the light degrees of freedom are simply the glueballs of the confining theory. Hence the throat does not allow embedding the SM degrees of freedom at the IR end. An attempt to construct throats with more structure at their bottom was carried out in [9], but involved complicated geometries whose holographic dual was really unknown.

Happily, new progress in understanding warped throats for other geometries generalising the conifold, as well as their interpretation in terms of duality cascades [10 − 12] and infrared confinement [12], allows to revisit these ideas. The general lesson is that warped

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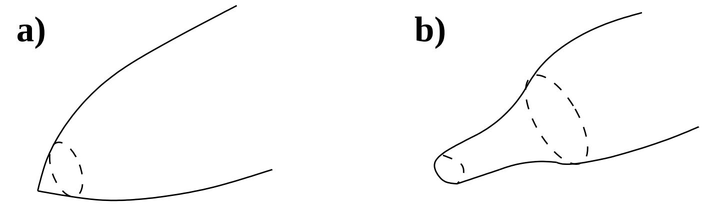

Figure 1: Throats with two different infrared behaviours: figure (a) shows a throat based on a geometry which upon complex deformation leaves a 'terminal' singularity. Figure (b) shows a throat based on a geometry which upon complex deformation leaves a geometry admitting a further complex deformation (in this case to a completely smooth space).

throats arise naturally by considering Calabi-Yau singularities which admit a complex deformation, corresponding to smoothing the singular point by growing a set of 3-cycles, on which to turn on fluxes. The holographic duals are given by duality cascades of the gauge theories arising on D3-branes at the singular geometries, much in the spirit of Vafa's brane-flux transitions [13].

This allows the description of warped throats, and their holographic duals, with richer infrared topology/dynamics. This paper is devoted to exploring some of the new model building possibilities of these constructions. In particular, an amusing feature is that the geometries after complex deformation may still contain a milder singularity. This is depicted in figure 1a. Locating D3-brane probes at such singularities corresponds to localising a chiral gauge sector at the IR brane of the warped throat construction. In the holographic picture this corresponds to a cascading gauge theory which after confinement leaves a set of light degrees of freedom whose effective field theory is a chiral gauge theory. This motivates our first application. We describe an explicit construction of a warped throat, and its holographic dual cascading theory, leading to a chiral gauge sector describing a 3-family SM-like chiral gauge theory. In 4d terms, it corresponds to a (supersymmetric) walking technicolor model, where all SM fields are composites of a confining theory, which is almost conformal in the ultraviolet, as we describe explicitly.

A second possibility is that the singularity at the bottom of the throat admits a further complex deformation, and hence an additional set of 3-cycles and fluxes. This corresponds to the development of a new throat whose UV is patched with the IR of the initial throat, as shown in figure 1b. The general case corresponds to a throat made up of several regions in the radial direction, corresponding to different warp factors, and separated by thin shells associated to the appearance of a 3-cycle supporting some flux [12]. In the holographic picture, this corresponds to the RG flow of a cascading gauge theory, which experiences several scales of partial confinement, after which the cascades of the remaining gauge theory follow. This provides a string theory description of RS constructions with several positive tension branes [14, 15]. Following [12] we describe one example of this kind, and point out possible applications of such multi-warp throats.

Our models should be regarded as illustrating the general techniques to construct throats suitable for model building applications, and their holographic dual field theories. Clearly, many other generalisations and extensions are possible. We expect much progress

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in model building applications of the new throats, and, although the constructions are supersymmetric (in order to automatically guarantee the stability of the throat, and to have control over the strongly coupled gauge theory dynamics), also in extending similar constructions in non-supersymmetric setups.

The paper is organised as follows. In section 2 we review the tools to describe singularities admitting complex deformations, in which the singularity is replaced by a set of localized 3-cycles. In section 3 we construct throats with singularities and chiral gauge sectors at its IR end, and their holographic duals. After a discussion of different interesting possibilities in section 3.1, we centre on a particular case in section 3.2. We describe the throat with a set of D3-branes at a $\mathbb{C}^3/\mathbb{Z}_3$ singularity at its bottom, and the holographic dual picture. In section 3.3 we improve the model by the addition of D7-branes in the throat picture, corresponding to adding flavours to the dual field theory. In section 4 we describe examples of multi-warp throats and possible applications. Section 5 contains our final remarks. Appendix A contains results for a local Calabi-Yau singularity related to that in section 3, and its field theory dual.

2. Complex deformed spaces

The natural way to construct warped throats with UV and IR cutoffs in string theory is via the introduction of 3-form fluxes, localized on a set of 3-cycles. The backreaction of the fluxes creates the warp factor. The compactification provides the UV cutoff, while the finite size of the 3-cycles cuts off the throats in the IR.

A natural way to obtain localized 3-cycles is to consider singular geometries and carry out a complex deformation. For instance, the deformed conifold arises this way, and so do other throats considered in [12]. Hence we are led to the study of complex deformations of local singularities.

In many interesting cases, these are simply described in terms of toric diagrams. For details we refer the reader to [16, 12]. For our purposes, it suffices to say that the geometries we consider can be encoded in web diagrams of segments on a 2-plane, carrying (p, q) labels, with the condition that the slope of the segment is related to the label, and that there is conservation of the label charge at junctions of segments [17]. The web encodes the geometry by specifying the locus on which certain $\mathbb{S}^1$ fibers degenerate [18, 16]. Interior segments and faces correspond to finite-size 2- and 4-cycles. The singular geometries where these shrink to zero size sometimes admits complex deformations, where finite-size 3-cycles appear. These are represented by a separation (in a new direction) of sub-webs in equilibrium. The 3-cycles can be encoded as segments joining the different sub-webs. The case of the conifold is shown in figure 2, and other examples will appear in the paper. See [12] for more examples.

Geometries with finite size 3-cycles can support 3-form fluxes. Specifically, one can turn on RR 3-form flux on the compact 3-cycles and NSNS 3-form flux on the dual (non-compact) 3-cycles. This leads to warped throats of the kind we would like to consider. The relation between the size of the 3-cycle and the warp factor with the fundamental parameters (the flux quanta) is exponential, leading to a hierarchy.

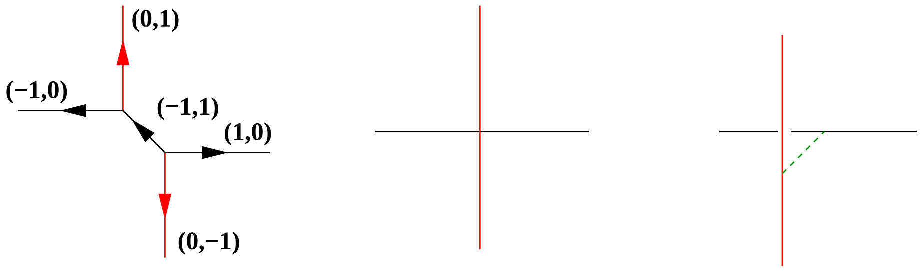

Figure 2: Conifold extremal transition. The finite segment in the first figure represents an S^2, while the dashed segment in the last figure corresponds to an S^3.

The bulk of these throats is approximately AdS_5, so they should admit a dual holographic description in terms of an approximately conformal field theory. Indeed throats arising from geometries using toric diagrams as above have a natural holographic interpretation. The dual corresponds to a duality cascade in a quiver gauge theory. The quiver gauge theory is given by the world-volume theory of D3-branes located at the singularity of the geometry in the singular limit (where 2/4-cycles and 3-cycles have zero size). The amount of flux is encoded in the presence of certain fractional branes in the dual, which in the regime of being much smaller than the total number of branes, leads to an approximately conformal theory. Along the duality cascade the gauge theory suffers a cascade of Seiberg dualities, and reduces its number of degrees of freedom. At the end of the cascade, there is strong gauge dynamics, confinement etc, with a characteristic scale holographically related to the size of the 3-cycle.

When there are several 3-cycles, the fluxes on them are independent. Hence one can obtain different sizes for the 3-cycles, with sizes hierarchically related to each other. This corresponds to a throat, at the bottom of which we trigger one complex deformation and are left over with a new, simpler geometry (which may subsequently suffer a further complex deformation etc). In the holographic picture, we have a field theory which cascades down until a strong dynamics scale. Below this scale, we have a new theory, which describes the left over geometry after deformation. If this new theory continues cascading, it corresponds to a further throat etc [12].

In the coming sections we describe the geometries (and holographic dual field theories) illustrating these two behaviours, namely throats admitting several complex deformations or throats ending up on singular geometries. In section 3 we describe a throat ending in a singularity at which we localise D-branes with the SM degrees of freedom. In section 4 we describe throats ending on configurations leading to further throats.

3. Singularities within throats

3.1 General discussion

A natural way to realize chiral gauge theories is with D3-branes at singularities [19, 20]. In fact, there are models reasonably similar to the SM with D3- and D7-branes at the $\mathbb{C}^3/\mathbb{Z}_3$ singularity [21].

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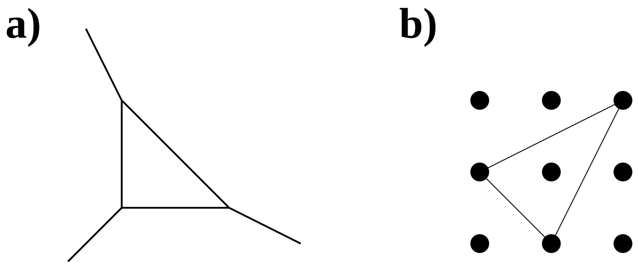

Figure 3: Web diagram and toric diagram for the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold.

So in order to obtain a chiral gauge sector from D3-branes at a singularity X, at the bottom of a warped throat, what we need is a geometry Y which admits a complex deformation, leaving X as the left over geometry. The 3-cycle associated to the complex deformation supports the flux creating the warped throat.

In what follows, we describe a technique to construct spaces Y which admit a complex deformation to a given singular space X. Although the procedure is general for toric varieties, for concreteness we centre on the case where the left over geometry X of interest is a $\mathbb{C}^3/\mathbb{Z}_3$ singularity. Other examples can be worked out similarly.

Hence we consider the geometry $\mathbb{C}^3/\mathbb{Z}_3$. This corresponds to the web diagram shown in figure 3a, and the toric diagram shown in figure 3b (for toric or grid diagrams, see [17]). Locating a set of D3-branes at this singular space leads to an $\mathcal{N}=1$ supersymmetric gauge theory with the following structure

$$\begin{array}{rl} \text{Vect. Mult.} & \mathrm{SU}(n)^3 \\ \text{Ch. Mult.} & 3(\Box,\overline{\Box},1)+3(1,\Box,\overline{\Box})+3(\overline{\Box},1,\Box)\,. \end{array} \tag{3.1}$$

We would like to consider spaces Y, which admit a complex deformation to X. This search is simpler if we require Y to be toric. Hence we need to look for web diagrams, which upon removal of a subweb in equilibrium leaves diagram 3a behind. Alternatively, webs for Y can be constructed by adding webs in equilibrium to figure 3a. It is important to realize that the final geometry of Y depends only on the asymptotic legs added, and not on details like the position of the new diagram etc. Some examples of such geometries are shown in figure 4.

Now turning on fluxes on the complex deformed space leads to a warped throat, cutoff in the infrared by the finite-size 3-cycle, that moreover contains a singularity locally of the form $\mathbb{C}^3/\mathbb{Z}_3$

It would be very useful to have a holographic description of these throats. This requires knowing the quiver gauge theory of D3-branes at the space Y, in the regime where it is singular. Unfortunately this information is available only for a few classes of singularities. These include the simplest singularities (conifold, suspended pinch point, etc) [7, 22, 23], complex cones over del Pezzo surfaces [24], and real cones over the $Y^{p,q}$ 5d horizons studied in [25]. Also, there are systematic tools to construct (and identify) the quiver field theories for orbifold quotients of any of the above geometries (and obviously of flat space).

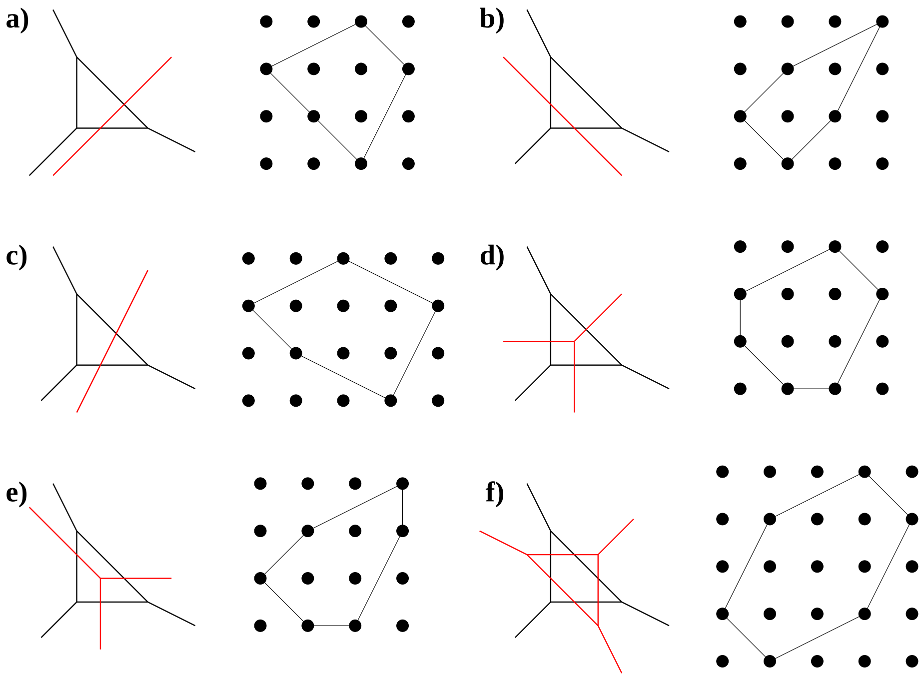

Figure 4: Web and toric diagrams for diverse examples of geometries Y which admit a complex deformation to the $\mathbb{C}^3/\mathbb{Z}_3$ singularity. The web diagrams are shown at the origin of the deformation branch, and the subwebs to be removed is shown in red.

As is illustrated by the examples in figure 4, the geometries get more complicated as the structure of the subweb to be removed becomes more involved. In general, the quiver gauge theory on D3-branes at such singularities is not known. However, there are a few special cases where it can be constructed. For instance, figure 4a corresponds to a quotient of the suspended pinch point, a singularity for which the quiver gauge theory is known. Similarly, figure 4f corresponds to a quotient of the cone over dP_3. In such cases, one can construct the quiver theories for the quotient space from the quiver theories for the parent space, in a systematic way. We will centre on one such example, to be discussed below. Nevertheless, we would like to emphasise that the general ideas are valid, even for geometries which are not orbifolds of simpler spaces, once the quiver gauge theories are known. In particular, some other geometries in figure 4 may allow the determination of the quiver gauge theory [26] by techniques of unHiggssing [27].

3.2 The orbifold of the SPP

Among the deformed spaces corresponding to the pictures in figure 4, we centre on that shown in figure 5a. This is because it corresponds to geometry for which the quiver gauge theory is easily obtained. Indeed, it corresponds to an orbifold of a singularity, the suspended pinch point (SPP), for which the field theory is known [22, 23]. In order to see it, we can consider the web diagram before the deformation, namely with finite size 2- and 4-cycles, shown in figure 5b. From this we can obtain the toric diagram, shown if figure 6.

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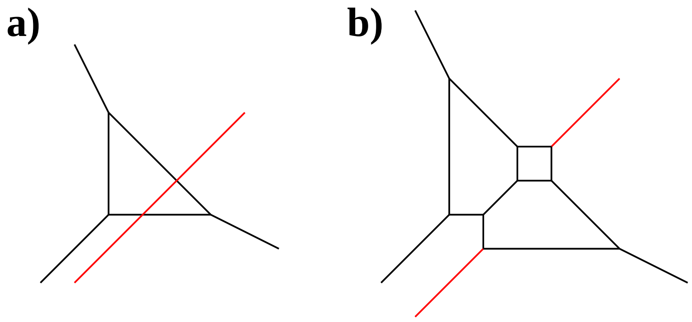

Figure 5: Web diagram for a geometry which admits a complex deformation to the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold. Figure (a) shows the geometry at the origin of the deformation branch, while figure (b) shows the geometry with finite size 2- and 4-cycles.

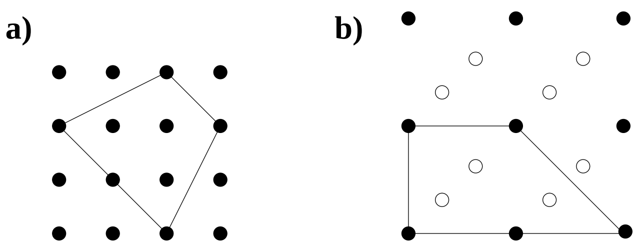

Figure 6: Toric diagram for the geometry in figure 5. By a linear transformation the toric diagram (figure (b)) manifestly shows that the geometry corresponds to a $\mathbb{Z}_3$ quotient of the suspended pinch point singularity. Namely, the black dots represent the toric diagram of the SPP geometry, and our geometry is obtained upon a refinement of the toric lattice, shown as open circles.

Using techniques in toric geometry, the diagram shows manifestly that the geometry corresponds to a $\mathbb{Z}_3$ quotient of the suspended pinch point singularity (SPP), see appendix A.4 for more details.

More explicitly, the SPP singularity can be described as the hypersurface in $\mathbb{C}^4$ given by the equation

$$xy - zw^2 = 0\,. \tag{3.2}$$

From the toric data it is possible to read off that our geometry corresponds to the quotient by the $\mathbb{Z}_3$ generated by[1]

$$x \to \alpha x\,, \qquad y \to \alpha y\,, \qquad z \to \alpha z\,, \qquad w \to \alpha^2 w \tag{3.3}$$

with $\alpha = e^{2\pi i/3}$. Notice that the $\mathbb{Z}_3$ action leaves invariant the holomorphic 3-form $\Omega = \frac{dxdydz}{zw}$ of the SPP, guaranteeing that the quotient is a new CY singularity.

[1]Although a systematic construction and identification of quotients in terms of toric data exists (see appendix A.4), there is a simple way to realize this is the right symmetry. It is the only $\mathbb{Z}_3$ symmetry of the SPP geometry which has the origin as the only fixed point, in agreement with the fact that the additional lattice points in 6b do not lie on the sides of the SPP polygon.

3.2.1 Geometry of the throat

The SPP geometry has a nice one-parameter complex deformation to a completely smooth space, as described in appendix A [12, 28]. This is given by modifying equation (3.2) to

$$xy - zw^2 = \epsilon w\,. \tag{3.4}$$

This is invariant under the $\mathbb{Z}_3$ action (3.3), hence the deformation is inherited to the quotient of the SPP. This corresponds precisely to the geometric separation of the line and the $\mathbb{C}^3/\mathbb{Z}_3$ sub-web in figure 5. The geometry contains a finite size 3-cycle (which is in fact a Lens space $\mathbb{S}^3/\mathbb{Z}_3$), which corresponds to the segment joining the two separated sub-webs. The left over geometry after the deformation contains a $\mathbb{C}^3/\mathbb{Z}_3$ singularity.

On general grounds it is expected that turning on M units of RR flux on the finite-size 3-cycles (as well as a suitable NSNS flux on its dual (non-compact) 3-cycle) leads to a warped throat. At its bottom, the throat is cut off by the finite-size 3-cycles, and it contains a $\mathbb{C}^3/\mathbb{Z}_3$ singularity. A chiral gauge sector at this bottom is obtained by introducing a small set of D3-branes at this singularity.[2] The latter are considered as probes, and do not modify the structure of the throat substantially.

The gauge theory on these branes thus leads to a chiral gauge theory

$$\begin{aligned} \text{Vect. mult.} \quad & \mathrm{U}(N) \times \mathrm{U}(N) \times \mathrm{U}(N) \\ \text{Ch. mult.} \quad & 3 \times [(N, \overline{N}, 1) + (1, N, \overline{N}) + (\overline{N}, 1, N)] \end{aligned} \tag{3.5}$$

where two of the U(1) factors are anomalous and become massive by the Green-Schwarz mechanism [29, 30].[3] Also, the ranks of the three gauge factors are equal due to tadpole/anomaly cancellation. Following [21], in section 3.3 we achieve non-equal ranks by introducing additional D7-branes in the configuration. Hence the above theory can be considered a toy model for the more realistic SM-like configurations to come.

The explicit metric for the deformed SPP is not known. Hence, it is difficult to be more precise about the detailed structure of the throat. Nevertheless from the geometric viewpoint, the size of the 3-cycle is stabilized by fluxes [3, 4]. A precise determination of this size in terms of the underlying parameters (the fluxes) would require determining the dependence of the periods of the holomorphic 3-form with the complex structure, around the point in moduli space where the size vanishes, as done for the deformed conifold in [4]. Even without this information, it is reasonable to expect the parametric dependence of the size, and the warp factor at the bottom of the throat, to be similar to the conifold case. Namely, the ratio of the warp factors at the tip, as compared with that at a radial distance r is $\simeq e^{K/Mg_s}$, where $N \simeq KM$ is the RR 5-form flux (and 5d cosmological constant) over the 5d horizon at the radial distance r.

[2]It is also possible to introduce anti-D3-branes, as we briefly comment in section 3.4, but we stick to the supersymmetric situation.

[3]The story for U(1)'s in systems of branes at singularities is general. All U(1) linear combinations except a diagonal one become massive due to $B \wedge F$ couplings. Hence one may work in the quiver theory without U(1)'s or keep them with the understanding that they eventually disappear. We work in these two pictures interchangeably.

3.2.2 Holographic field theory dual

The field theory. The holographic dual description of the above throat is in terms of the gauge field theory of D3-branes at our space Y, in the singular limit. In order to construct it, the most efficient way is to exploit its realization at a $\mathbb{Z}_3$ quotient of the SPP quiver gauge theory, following ideas in [23].

The gauge group on the world-volume of D3-branes at the SPP singularity is $\mathrm{U}(N_1) \times \mathrm{U}(N_2) \times \mathrm{U}(N_3)$, with the chiral multiplet content given by:

	$\mathrm{U}(N_1)$	$\mathrm{U}(N_2)$	$\mathrm{U}(N_3)$
F	$\square$	$\overline{\square}$	1
$\widetilde{F}$	$\overline{\square}$	$\square$	1
G	1	$\square$	$\overline{\square}$
$\widetilde{G}$	1	$\overline{\square}$	$\square$
H	$\overline{\square}$	1	$\square$
$\widetilde{H}$	$\square$	1	$\overline{\square}$
Φ	Adj.	1	1

The ranks are unconstrained by anomaly cancellation, corresponding to the possibility of introducing fractional D-branes in the system.

In order to identify the $\mathbb{Z}_3$ action on the above fields, we note that the action (3.3) on x, y, z, w corresponds to the action

$$\begin{aligned} &F \to \alpha^2 F\,; \qquad \widetilde{F} \to \widetilde{F}\,: \qquad G \to G\,; \qquad \widetilde{G} \to \alpha\widetilde{G} \\ &H \to \alpha^2 H\,; \qquad \widetilde{H} \to \widetilde{H}\,; \qquad \Phi \to \alpha\Phi \end{aligned} \tag{3.6}$$

This is obtained as the $\mathbb{Z}_3$ symmetry of the theory which agrees with the action (3.3) upon use of the relations (A.4) (arising from the realization of the SPP geometry as the moduli space of the gauge theory). Alternatively it can be obtained explicitly by using the toric techniques in appendix A.4.

In addition, we need to specify the $\mathbb{Z}_3$ action on the gauge (Chan-Paton) degrees of freedom. This is done by choosing three commuting $\mathrm{U}(n_i)$ gauge transformations, which without loss of generality can be parametrised as the diagonal matrices

$$\begin{aligned} \gamma_{\theta,1} &= \mathrm{diag}\,(\mathbb{1}_{m_0}, \alpha\mathbb{1}_{m_1}, \alpha^2\mathbb{1}_{m_2}) \\ \gamma_{\theta,2} &= \mathrm{diag}\,(\mathbb{1}_{n_0}, \alpha\mathbb{1}_{n_1}, \alpha^2\mathbb{1}_{n_2}) \\ \gamma_{\theta,3} &= \mathrm{diag}\,(\mathbb{1}_{p_0}, \alpha\mathbb{1}_{p_1}, \alpha^2\mathbb{1}_{p_2})\,. \end{aligned} \tag{3.7}$$

In order to describe the field theory on D3-branes at the quotient space, we project the field theory of the SPP onto states invariant under the combined (geometric plus Chan-Paton) $\mathbb{Z}_3$ action.

Regarding the different fields as matrices, the projection conditions on the vector multiplets V_i, and the chiral multiplets are given by

$$\begin{aligned} &V_i = \gamma_{\theta,i} V_i \gamma_{\theta,i}^{-1}\,; \qquad \Phi = \alpha\gamma_{\theta,1}\Phi\gamma_{\theta,1}^{-1}\,; \qquad F = \alpha^2\gamma_{\theta,1} F \gamma_{\theta,2}^{-1}\,; \qquad \widetilde{F} = \gamma_{\theta,2}\widetilde{F}\gamma_{\theta,1}^{-1} \\ &G = \gamma_{\theta,2} G \gamma_{\theta,3}^{-1}\,; \qquad \widetilde{G} = \alpha\gamma_{\theta,3}\widetilde{G}\gamma_{\theta,2}^{-1}\,; \qquad H = \alpha^2\gamma_{\theta,3} H \gamma_{\theta,1}^{-1}\,; \qquad \widetilde{H} = \gamma_{\theta,1}\widetilde{H}\gamma_{\theta,3}^{-1}\,. \end{aligned} \tag{3.8}$$

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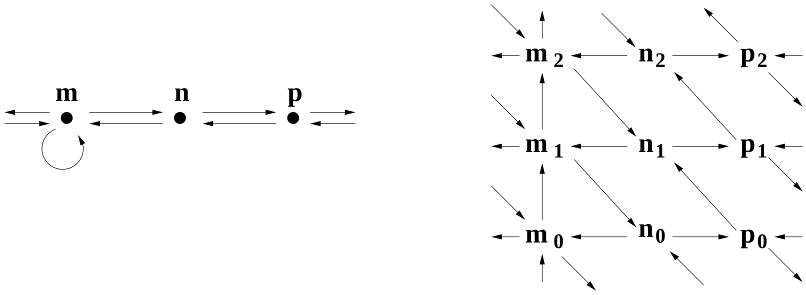

Figure 7: The quiver for the SPP and its orbifold. The SPP quiver is understood to be periodically identified along the horizontal direction, while that of its orbifold should be identified along the horizontal and vertical directions.

The quiver gauge theory gauge group is $\prod_i (\mathrm{U}(m_i)\times\mathrm{U}(n_i)\times\mathrm{U}(p_i))$, and the chiral multiplet content is given by

$$\begin{array}{rl} & \mathrm{U}(m_i)\times\mathrm{U}(n_i)\times\mathrm{U}(p_i) \\ F_{i,i-1} & (m_i, \overline{n}_{i-1}) \\ \widetilde{F}_{i,i} & (n_i, \overline{m}_i) \\ G_{i,i} & (n_i, \overline{p}_i) \\ \widetilde{G}_{i,i+1} & (p_i, \overline{n}_{i+1}) \\ H_{i,i-1} & (p_i, \overline{m}_{i-1}) \\ \widetilde{H}_{i,i} & (m_i, \overline{p}_i) \\ \Phi_{i,i+1} & (m_i, \overline{m}_{i+1})\,. \end{array} \tag{3.9}$$

With this notation, the superpotential reads

$$\begin{aligned} W = &\ \widetilde{F}_{i,i}F_{i,i-1}G_{i-1,i-1}\widetilde{G}_{i-1,i} - \widetilde{G}_{i,i+1}G_{i+1,i+1}H_{i+1,i}\widetilde{H}_{i,i} + \\ & + \widetilde{H}_{i,i}H_{i,i-1}\Phi_{i-1,i} - \Phi_{i,i+1}F_{i+1,i}\widetilde{F}_{i,i}\,. \end{aligned} \tag{3.10}$$

The quiver for the spp and its orbifold are shown in figure 7.

The realization of our geometry as a quotient of the SPP is a useful tool in this example, both in the construction of the quiver, and in the subsequent analysis of the field theory. However, we prefer to carry out the remaining analysis of the field theory without using this information, in order to illustrate that the holographic dual cascade etc can be constructed once the quiver gauge theory is known, even for examples which are not orbifolds. The relation to similar phenomena in the parent theory is described in appendix A.

The duality cascade. Imposing the conditions of cancellation of anomalies, the ranks of the nine groups in the quiver are partially constrained. In the absence of further branes in the configuration (e.g. D7-branes like in section 3.3) the most general rank assignment is

$$m_i = N\,, \qquad n_i = N+M\,, \qquad p_i = N+P\,. \tag{3.11}$$

This shows that the configuration can support two kinds of fractional branes. In the following we study the behaviour of the field theory for $P=0$, and show that it leads to a duality cascade. The whole analysis may be regarded as a $\mathbb{Z}_3$ quotient of that in

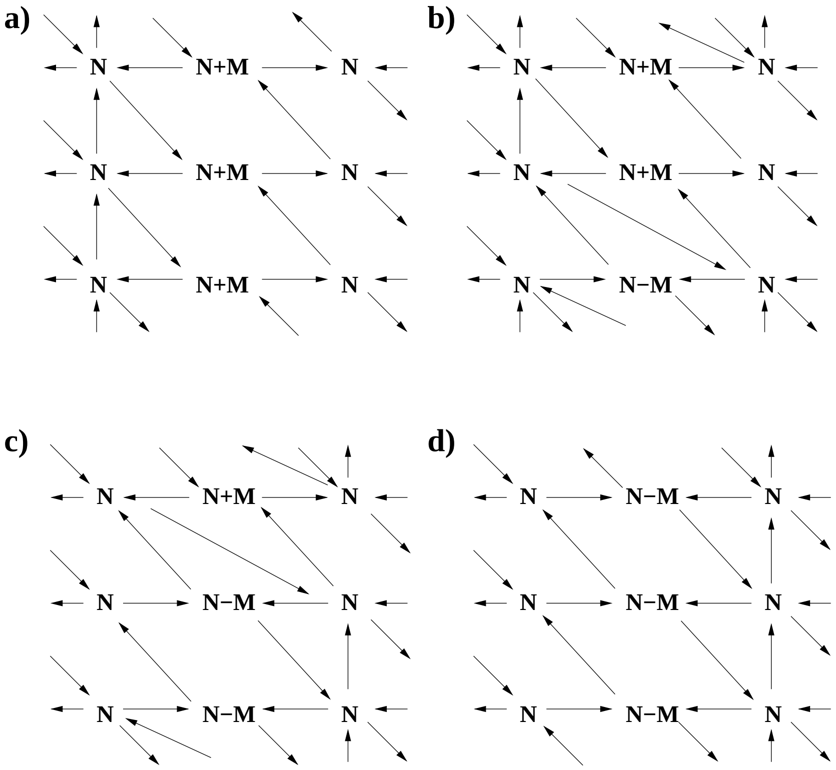

Figure 8: The quivers in the sequential Seiberg dualities. The diagrams are to be periodically identified along the horizontal and vertical directions. Figure (a) shows the initial quiver for the orbifold of the SPP. Figure (b) shows the quiver after dualising the node n_0, figure (c) shows the quiver after dualising the node n_1 and figure (d) shows the quiver after dualising n_2. In each dualization, the dualised node ends up with rank $N - M$.

section A.2. but we repeat it on its own, since it illustrates the general technique, valid for more general examples.

We consider the situation where $m_i = N$, $n_i = N + M$, $p_i = N$. We also consider the UV couplings of groups in the same row to be equal. In this situation, when the theory flows to the IR we expect the nodes with rank $N + M$ to become strongly coupled and we should Seiberg dualise them. This may be done sequentially, namely we first dualise n_0, then n_1 and finally n_2. As described in [31] (see also [32]), dualization of a node simply amounts to simple operations with the arrows of the quiver. Namely, one reverses all arrows with one endpoint in the dualised node (replaces quarks by dual quarks), and adds arrows corresponding to the composition of ingoing and outgoing arrows (introduces the mesons). In addition, one should replace the number of colors by its dual ($N_c' = N_f - N_c$) and removes paired arrows (from mass terms in the superpotential). The whole process of the three Seiberg dualities is shown in figure 8.

The final result is the quiver shown in figure 8d. It is a quiver of the same kind as the original one, up to a translation of the nodes (of two nodes to the left in the horizontal direction), and a change of ranks. This facilitates the next dualization steps, which follow an analogous pattern.

Following the flow to the infrared, the next nodes in becoming strongly coupled are the three nodes of type m (first column), which should be simultaneously dualised. Then one should dualise nodes of type p, then n again, and continue dualising nodes of type m, p and n again. After this the theory comes back to exactly the original one, with the same value of M but with an effective N of $N - 6M$. This completes a duality cycle, whose repeated occurrence in the RG flow originates the duality cascade.

In section A.2 we compare this flow with that for the SPP parent theory with a similar assignment of fractional branes. The above flow amounts to simply a $\mathbb{Z}_3$-projected version of the duality cascade of the SPP, studied in [12]. This relation guarantees even more strongly that the above duality cascade in the orbifold theory takes place in the RG flow of the theory, at least for a general $\mathbb{Z}_3$-invariant choice of UV gauge couplings.

The infrared deformation. The duality cascade proceeds until the effective number of D3-branes is comparable with the number of fractional branes M. In this situation, strong infrared dynamics takes over and ends the cascading behaviour. For instance, for N a multiple of M the final gauge theory simply contains three decoupled $\mathrm{SU}(M)$ SYM-like theories, which have a common dynamical scale Λ, at which confinement occurs. In the supergravity dual, the warped throat describing the cascade is cutoff in the infrared by a complex deformation of the geometry, namely by a finite size 3-cycle, to a $\mathbb{C}^3/\mathbb{Z}_3$.

A simple way to recover this deformation from the field theory viewpoint is to consider introducing M additional D3-branes probing the infrared theory, and to study their moduli space (which corresponds to probing the geometry of the supergravity dual). Hence, following [12], we are led to studying the quiver gauge theory with $m_i = M$, $n_i = 2M$, $p_i = M$.

In this situation, the nodes $\mathrm{SU}(2M)$ have the same number of colors and flavours. Hence, ignoring the $\mathrm{SU}(M)$ dynamics (whose coupling is smaller, hence may be regarded as global symmetries), these gauge factors develop a quantum deformation of their moduli space. Specifically, for the i^{th} $\mathrm{SU}(2M)_i$ factor we introduce the mesons

$$\mathcal{M}_i = \begin{bmatrix} A_{i+1,i} & B_{i+1,i} \\ C_{i-1,i} & D_{i-1,i} \end{bmatrix} = \begin{bmatrix} F_{i+1,i}G_{i,i} & F_{i+1,i}\widetilde{F}_{i,i} \\ \widetilde{G}_{i-1,i}G_{i,i} & \widetilde{G}_{i-1,i}\widetilde{F}_{i,i} \end{bmatrix} \tag{3.12}$$

and the baryons, schematically $\mathcal{B}_i$, $\widetilde{\mathcal{B}}_i$. These are the infrared degrees of freedom after confinement of the corresponding gauge factor. They are subject to the quantum modified constraint

$$\det \mathcal{M}_i - \mathcal{B}_i\widetilde{\mathcal{B}}_i = \Lambda^{4M} \tag{3.13}$$

where Λ is a strong coupling dynamics scale. These constraints can be implemented in the superpotential via Lagrange multipliers X_i. The superpotential thus reads

$$\begin{aligned} W = {} & D_{i-1,i}A_{i,i-1} - C_{i,i+1}H_{i+1,i}\widetilde{H}_{i,i} + \widetilde{H}_{i,i}H_{i,i-1}\Phi_{i-1,i} - \\ & -\Phi_{i,i+1}B_{i+1,i} - X_i(\det \mathcal{M}_i - \mathcal{B}_i\widetilde{\mathcal{B}}_i)\,. \end{aligned} \tag{3.14}$$

The dynamics of the D3-brane probes is manifest along the mesonic branch, where the quantum constraint is saturated with vevs for the mesonic operators, namely

$$X_i = 1\,; \qquad \mathcal{B}_i = \widetilde{\mathcal{B}}_i = 0\,; \qquad \det \mathcal{M}_i = \Lambda^{4M}\,. \tag{3.15}$$

Along this mesonic branch gauge symmetry is broken. The maximally unbroken symmetry follows e.g. for $\det \mathcal{M}_i \propto \mathbb{1}_{2M}$ where the breaking pattern is (keeping the rank notation general, for the sake of clarity)

$$\mathrm{U}(m_0) \times \mathrm{U}(p_2) \to \mathrm{U}(N_0)\,; \qquad \mathrm{U}(m_1) \times \mathrm{U}(p_0) \to \mathrm{U}(N_1)\,; \qquad \mathrm{U}(m_2) \times \mathrm{U}(p_1) \to \mathrm{U}(N_2)\,. \tag{3.16}$$

As discussed in [12], it is permissible (and even convenient for future discussion) to include the U(1) centre of mass factors in the discussion.

With respect to the symmetry (3.16), the quantum numbers for the matter multiplets are:

$$\begin{array}{rlrlrl} A_{02}, D_{20}\colon & (N_0, \overline{N}_0) & A_{10},\, D_{01}\colon & (N_1, \overline{N}_1) & A_{21},\, D_{12}\colon & (N_2, \overline{N}_2) \\ B_{10}\colon & (N_1, \overline{N}_0) & B_{21}\colon & (N_2, \overline{N}_1) & B_{02}\colon & (N_0, \overline{N}_2) \\ C_{20}, H_{21}, \widetilde{H}_{00}, \Phi_{01}\colon & (N_0, \overline{N}_1) & C_{01}, H_{02}, \widetilde{H}_{11}, \Phi_{12}\colon & (N_1, \overline{N}_2) & C_{12}, H_{10}, \widetilde{H}_{22}, \Phi_{20}\colon & (N_2, \overline{N}_0) \end{array}$$

Restricting to the abelian case, and along the mesonic branch, the superpotential reads

$$\begin{aligned} W = {} & D_{i-1,i}A_{i,i-1} - C_{i,i+1}H_{i+1,i}\widetilde{H}_{i,i} + \widetilde{H}_{i,i}H_{i,i-1}\Phi_{i-1,i} - \\ & -\Phi_{i,i+1}B_{i+1,i} - A_{i+1,i}D_{i-1,i} + C_{i-1,i}B_{i+1,i}\,. \end{aligned} \tag{3.17}$$

Using the equations of motion for the D, A and B fields, we obtain

$$\begin{aligned} A_{0,2} &= A_{1,0} = A_{2,1} \equiv A \\ D_{0,1} &= D_{1,2} = D_{2,0} \equiv D \\ \Phi_{1,2} &= C_{0,1}\,, \qquad \Phi_{2,0} = C_{1,2}\,, \qquad \Phi_{0,1} = C_{2,0}\,. \end{aligned} \tag{3.18}$$

Essentially, the fields B become massive with the fields $\Phi - C$, while overall combinations of the fields A, B, $\Phi + C$, D remain light. They are subject to the quantum constraint, and describe the motion of the probes in a complex deformed geometry.

The left over geometry is encoded in the left over field theory. It is useful to relabel the fields as $C_{i-1,i} = X_{i,i+1}$, $H_{i,i-1} = Y_{i-2,i-1}$, $\widetilde{H}_{i,i} = Z_{i,i+1}$. The field theory has a gauge group $\mathrm{U}(N_0) \times \mathrm{U}(N_1) \times \mathrm{U}(N_2)$, and chiral multiplets $X_{i,i+1}$, $Y_{i,i+1}$, $Z_{i,i+1}$ in the representation $(N_i, \overline{N}_{i+1})$. Namely

$$3 \times [(N_0, \overline{N}_1) + (N_1, \overline{N}_2) + (N_2, \overline{N}_0)]\,. \tag{3.19}$$

Using (3.18) the superpotential reads

$$W = -X_{i,i+1}Y_{i+1,i+2}Z_{i+2,i} + X_{i,i+1}Z_{i+1,i+2}Y_{i+2,i}\,. \tag{3.20}$$

This is precisely the field theory of D3-branes at the $\mathbb{C}^3/\mathbb{Z}_3$ singularity. Hence the latter is the left over geometry after the complex deformation, namely the structure of the geometry at the end of the dual warped throat is a $\mathbb{Z}_3$ orbifold singularity.

Using this information, and following similar discussions of the conifold case in [5] and appendix A for the SPP, we may consider the field theory cascade with $N = kM + P$, with $k \in \mathbb{Z}$ and $0 \leq P \leq M$. Again the theory suffers a duality cascade and infrared confinement, leaving an $\mathrm{U}(P)^3$ theory in the infrared. This can be obtained by analysing the next to last step in the cascade, where the ranks are $m_i = P$, $n_i = M + P$, $p_i = P$. The strong $\mathrm{SU}(M + P)$ dynamics confines and generates an Affleck-Dine-Seiberg superpotential [33]. This forces the meson determinant to acquire a non-zero vev, which triggers a breaking of the symmetry and a collapse of the quiver theory exactly as in the above analysis. The left over theory indeed describes a $\mathrm{U}(P)^3$ gauge theory associated to the quiver of P D3-branes at the $\mathbb{C}^3/\mathbb{Z}_3$ singularity. In the dual supergravity description, we have a throat based on the quotient of the SPP geometry, which is cutoff in the infrared by a 3-cycle which corresponds to the complex deformation to the $\mathbb{C}^3/\mathbb{Z}_3$ singularity, at which we have P explicit D3-brane probes. In heuristic terms the dynamics of the kM D3-branes create the complex deformation, while the additional P D3-branes remain as probes of the left over $\mathbb{Z}_3$ orbifold singularity.

Considering this chiral gauge theory as a momentarily reasonable toy model for particle physics, we have provided a purely 4d field theory interpretation of the stabilization of the hierarchy, and the appearance of the SM fields in the infrared. The Standard Model fields (gauge bosons, chiral fields, and Higgsses) are composites of a larger gauge theory at higher energies, which is almost conformal in large ranges of energies, but eventually confines at a scale (which one should consider close to the TeV). The infrared dynamics of this theory is such that (the above toy version of) the SM arises as the corresponding composite gauge bosons, and mesonic degrees of freedom. Thus the hierarchy is generated by a peculiar version of walking technicolor. It is worthwhile to make two comments on the peculiarity of this theory: a first unusual feature of this model is that the SM gauge bosons are also composites, and arise only as infrared excitations. A second one is the cascading structure of the UV theory, with different effective theories in different energy ranges, each of them being described in terms of degrees of freedom which are composite in terms of the theory at immediately higher energies. Moreover, there is no energetic regime in which any of these degrees of freedom are weakly coupled (notice that there is no UV asymptotic freedom in the theory), hence the main signature at high energies is the appearance of glueballs of the confining theories at energies immediately above the SM. Note that in the supergravity picture these glueballs are the Kaluza-Klein excitations of the graviton.

Although more involved (and realistic) models will be described in coming sections, the basic physical ideas about the 4d interpretation of the SM remain valid in such more general configurations.

3.3 Adding D7-branes

We have succeeded in constructing a throat with a chiral D-brane sector at its tip, and with a tractable holographic dual. However, D-brane configurations leading to theories with SM-like spectrum require additional ingredients, like D7-branes [21, 9]. In this section we describe the introduction of D7-branes in the warped throat picture, as well as the corresponding holographic field theory description.

As discussed in [34] and below, the D7-branes may be treated in the probe approximation as long as their number is much smaller than the fluxes. Also, in order to preserve supersymmetry,[4] the 4-cycle must be holomorphic.

[4]In the presence of fluxes, the same condition ensures that the 4-cycle is a generalized calibration, and

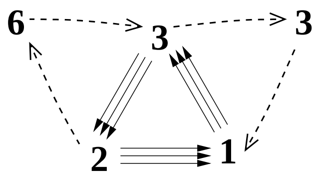

Figure 9: Quiver for the gauge theory at the infrared of the cascading throat. It is a 3-family SM-like theory.

We would like to consider a set of D7-branes, spanning a (non-compact) 4-cycle passing through the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold singularity. These may be obtained by considering D7-branes in the SPP geometry, wrapped on a 4-cycle passing through the $\mathbb{Z}_3$ fixed point $x = y = z = w = 0$, and performing the $\mathbb{Z}_3$ quotient. This 4-cycles should be holomorphic, namely, defined by a holomorphic equation in the complex variables x, y, z, w.

Again, the difficulty lies in identifying the open string degrees of freedom arising from this new ingredient. This information may be obtained by starting with D3/D7-brane systems at $\mathbb{C}^2/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ and carrying out a partial resolution to the SPP. Fortunately this kind of exercise has been carried out (in the presence of additional orientifold projections) in [36], hence we may extract enough information for our purposes from this reference. In this section we consider the particular example, enough for our purposes, of a stack of D7-branes wrapped on the 4-cycle $w = 0$.[5]

3.3.1 The standard model on the throat

We consider the throat we had before, namely, 3-form fluxes on the deformation of the $\mathbb{Z}_3$ quotient of the SPP to the $\mathbb{Z}_3$ singularity (equivalently, the $\mathbb{Z}_3$ quotient of $xy - zw^2 = \epsilon w$). We also choose the RR flux M much larger than the number of branes to be introduced below, so that the latter can be treated as probes.

At the singularity, we would like to consider a system of D3- and D7-brane probes leading to the semi-realistic models in [21], in analogy with [9]. This is achieved by introducing six D3-brane probes at the $\mathbb{Z}_3$ singularity, and a number of D7-branes wrapped on $w = 0$. In order to obtain a realistic spectrum, we make the following choice of Chan-Paton matrices

$$\begin{aligned} \gamma_{\theta,3} &= \mathrm{diag}\,(\mathbb{1}_3, \alpha\mathbb{1}_2, \alpha^2\mathbb{1}_1) \\ \gamma_{\theta,7} &= \mathrm{diag}\,(\alpha\mathbb{1}_3, \alpha^2\mathbb{1}_6) \end{aligned} \tag{3.21}$$

which is consistent with cancellation of RR tadpoles.

Since the model is locally $\mathbb{C}^3/\mathbb{Z}_3$, local properties of the configuration (like the 33 and 37 spectrum) can be analysed ignoring the fluxes and the global geometry. The local

thus that the brane preserves supersymmetry [35].

[5]From the equations of the SPP, this implies z arbitrary and $xy = 0$. Hence the 4-cycle is reducible in two components, corresponding to $x = 0$, y arbitrary and $y = 0$, x arbitrary. The matter content described in section 3.3.2 corresponds to D7-branes in one of these component 4-cycles, while for D7-branes in the other 4-cycle leads to a different matter content, which incidentally appears in a Seiberg dual version along the cascade, see figure 12*c*.

Matter fields	Q_3	Q_2	Q_1	$Q_{\mathrm{U}(3)}$	$Q_{\mathrm{U}(6)}$	Y
33 sector						
$3(3,2)$	1	-1	0	0	0	1/6
$3(\bar{3},1)$	-1	0	1	0	0	-2/3
$3(1,2)$	0	1	-1	0	0	1/2
37 sector						
$(3,1;\overline{3},1)$	1	0	0	-1	0	-1/3
$(\bar{3},1;1,6)$	-1	0	0	0	1	1/3
$(1,2;1,\overline{6})$	0	1	0	0	-1	-1/2
$(1,1;3,1)$	0	0	-1	1	0	1

Table 1: Spectrum of $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ model. We present the quantum numbers under the $\mathrm{U}(1)^5$ groups. The first three U(1)'s come from the D3-brane sector. The next two come from the D7-brane sectors. The last column shows charges under the D3-brane massless U(1) linear combination, which plays the role of hypercharge.

analysis thus follows [21]. As discussed in [21], a single linear combination of the D3-brane U(1)s remains light, and plays the role of hypercharge. The group on the D7-branes leads to 8d gauge dynamics, which should be considered as a global symmetry from the D3-brane viewpoint. Upon embedding the throat in a global compactification, the D7-brane gauge dynamics becomes 4d, but the unbroken 4d gauge group and matter content depend on the compactification boundary conditions and are model-dependent. They will not be discussed in our local approach. The spectrum of the model is shown in table 1, and corresponds to the quiver diagram 9.

The above construction realizes a configuration of D-branes leading to an interesting chiral gauge sector localized at the IR end of a warped throat. The construction is supersymmetric, although one may attempt to do non-supersymmetric model building.

3.3.2 Holography and the standard model

We now turn to a description of the holographic dual of the above configuration. General results in the gauge/gravity correspondence imply that the addition of D7-branes in the gravity side must correspond to the addition of flavours in the holographic dual quiver gauge theory. Namely, fields in the (anti)fundamental representation of some gauge factors of the field theory, and transforming under the global symmetry groups associated to the D7-branes. Since the geometry is unchanged, we expect a duality cascade of the D3-brane quiver gauge theory, with structure given as above (section 3.2.2). More specifically, the N_f D7-branes in the gravity side are treated in the probe approximation, namely at zeroth order N_f/N, N_f/M. As discussed in [34], this corresponds to the quenched approximation, in which the effect of the flavours is ignored in the gauge dynamics. Hence the gauge dynamics reduces to the pure D3-brane case.

Notice that nevertheless, as we describe below, the field theory analysis can be carried out even including these effects. This would be particularly relevant in the infrared in situations where M is not particularly large.

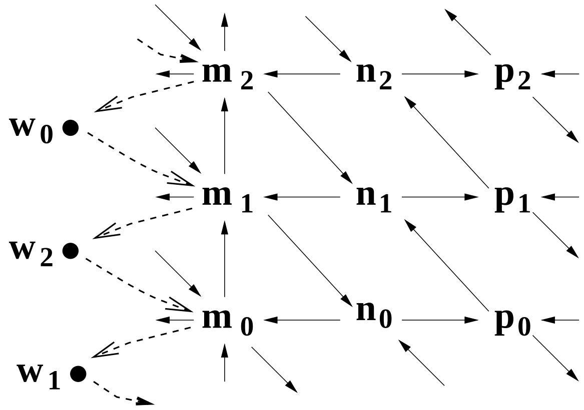

Figure 10: Extended quiver diagram for D3- and D7-branes at the quotient of the SPP.

The field theory. In the following we discuss the field theory. In order to determine the 37 fields in the final quiver field theory, we describe them in the SPP field theory and carry out the $\mathbb{Z}_3$ quotient.

The spectrum of 37 strings for the D7-branes on $w = 0$ in the SPP can be extracted from [36]. Before the orbifold projection, the 3-7 open strings lead to two $\mathcal{N} = 1$ chiral multiplets Q, $\widetilde{Q}$ in the representations $(N_{D3}; \overline{N}_{D7})$ and $(\overline{N}_{D3}, N_{D7})$, where the first entry corresponds to the D3-brane gauge factor with adjoints, and the second to the $\mathrm{U}(N_{D7})$ on the D7-branes. There is also a superpotential

$$W = \Phi Q \widetilde{Q} \,. \tag{3.22}$$

It is now easy to quotient the configuration by the $\mathbb{Z}_3$ symmetry, and obtain the effect of the new D7-branes on the quotient field theory. Since the D7-branes are mapped to themselves by the $\mathbb{Z}_3$ action, we introduce a general Chan-Paton action

$$\gamma_{\theta,D7} = \mathrm{diag}\,(\mathbb{1}_{w_0}, \alpha \mathbb{1}_{w_1}, \alpha^2 \mathbb{1}_{w_2})\,. \tag{3.23}$$

The geometric action on Q, $\widetilde{Q}$ may be obtained from the behaviour of the vertex operators for the corresponding open strings. The result is $Q \to \alpha Q$, $\widetilde{Q} \to \alpha \widetilde{Q}$, consistently with the $\mathbb{Z}_3$ invariance of the superpotential (3.22). Carrying out the $\mathbb{Z}_3$ projection, the 3-7 spectrum in the quotient of the SPP is

$$\text{Ch. Mult.} \quad \sum_i (m_i; \overline{w}_{i+1}) + (\overline{m}_{i+1}; w_i)\,. \tag{3.24}$$

This may be encoded in an extended quiver diagram [19, 37], with nodes associated to the D7-brane groups, and arrows representing the 3-7 and 7-3 states. This is shown in figure 10.

For notational convenience, we describe the rank assignments in a quiver like figure 10 as a matrix

$$\begin{pmatrix} w_0 & | & m_2 & n_2 & p_2 \\ w_2 & | & m_1 & n_1 & p_1 \\ w_1 & | & m_0 & n_0 & p_0 \end{pmatrix}. \tag{3.25}$$

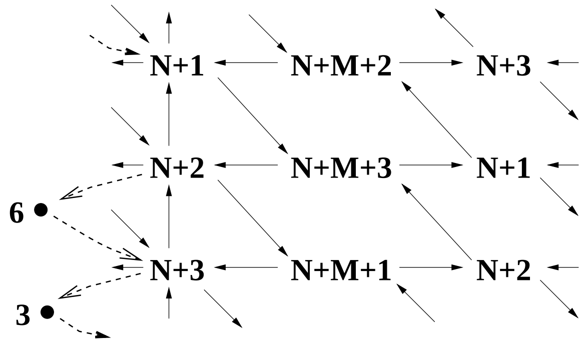

Figure 11: Quiver for the rank assignments leading to the duality cascade with flavours.

Similarly to the flat space orbifold case, the addition of D7-branes modifies the RR tadpole cancellation conditions. They can be recovered as the conditions of cancellation of non-abelian gauge anomalies [38], and will not be further discussed.

In order to describe the holographic dual of the throat with the above assignment of D-branes at its tip, we should realize that the D7-branes correspond to a global symmetry group, and hence they are unchanged by the gauge dynamics. Hence, the choice of ranks should agree with the fractional D7-branes on the gravity side, namely $w_0 = 0$, $w_1 = 3$, $w_2 = 6$. With this choice, the general anomaly free (i.e. RR tadpole free) rank assignment for the D3-brane nodes corresponds to the matrix

$$\begin{pmatrix} 0 & | & N_1+1 & N_2+2 & N_3+3 \\ 6 & | & N_1+2 & N_2+3 & N_3+1 \\ 3 & | & N_1+3 & N_2+1 & N_3+2 \end{pmatrix}. \tag{3.26}$$

Clearly, in order to recover the correct dual gravitational background, we are led to the choice $N_1 = N_3 = N$, $N_2 = N + M$, with $N \gg M$. This is shown in figure 11

Denoting N_f the generic number of flavours, the effect of flavours in the field theory is non-trivial, due to the modification of RR tadpole conditions, but subleading in N_f/N. Hence we expect to recover the right structure in the quenched approximation.

The duality cascade. The above theory has a duality cascade, which at zeroth order in N_f/M reduces to the pure D3-brane cascade. However, there are non-trivial effects of the flavours, which we would like to briefly mention. These involve changes in the ranks of the dual gauge factors, due to the additional flavours, and the appearance of additional meson degrees of freedom.[6] Several steps in the duality cascade are shown in figure 12. As is manifest by iterating the duality sequence, the duality cascade eventually comes back to the original quiver, but with an assignment of fractional branes differing from the original one in amounts of order one. Hence, the cascade is not exactly periodic in the presence of flavours. Notice however that these effects are subleading in the quenched approximation, hence the geometry in the dual is unchanged with respect to the case without flavours. In fact,

[6]These effects are crucial in keeping the dual theories anomaly free. In the quenched approximation, some intermediate steps in the duality cycle describe anomalous theories, but with anomalies being subleading in N_f/N.

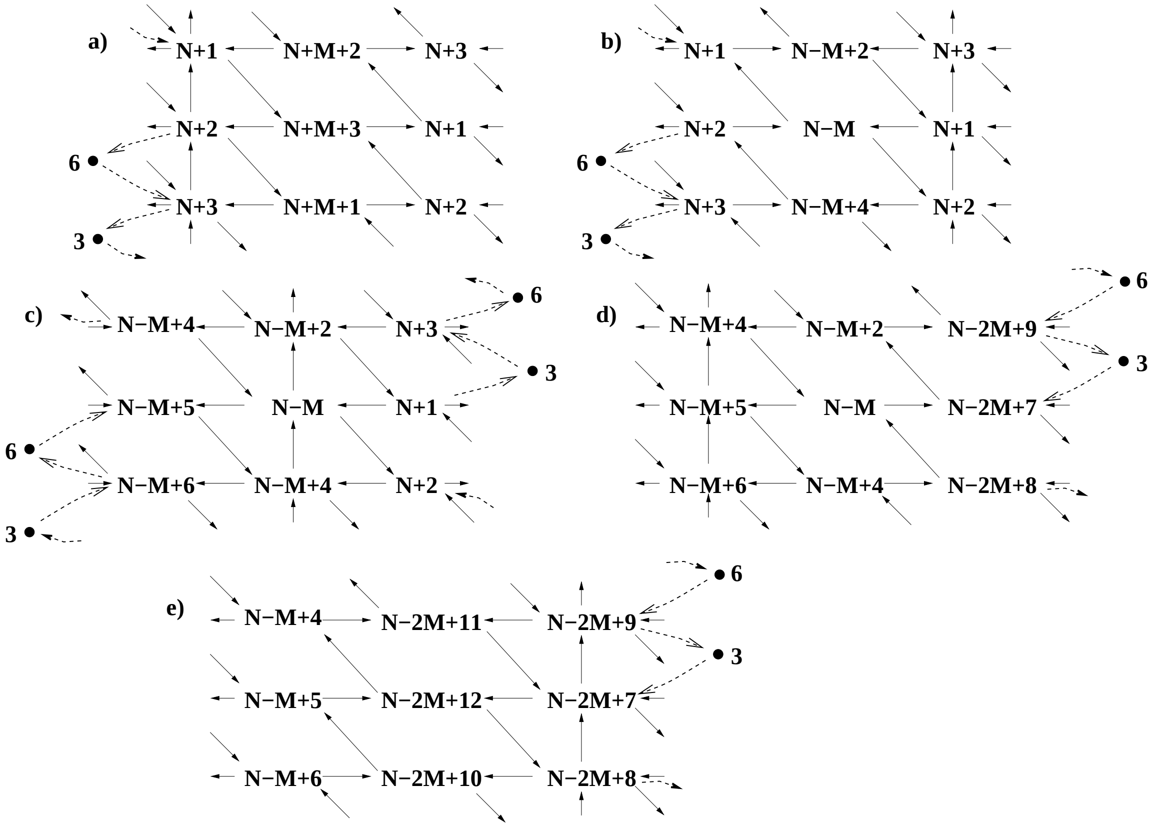

Figure 12: Quivers diagrams for some steps in the duality cascade of the theory with flavours. For clarity, in figure (c) the D7-brane nodes appear replicated.

this discussion has been carried in detail for the case of the conifold with flavours in [39], where moreover it was shown that field theory N_f/N effects are matched by the leading N_f/N backreaction of the D7-brane on the background. Clearly, our above throat with the D7-brane regarded as a probe should match the cascade in the quenched approximation.

The infrared deformation. The duality cascade ends when the number of D3-branes is exhausted. By tuning the choice of UV rank assignments, one may arrange the corresponding quiver gauge theory to correspond to the ranks

$$\begin{pmatrix} 0 & | & 1 & M+2 & 3 \\ 6 & | & 2 & M+3 & 1 \\ 3 & | & 3 & M+1 & 2 \end{pmatrix}. \tag{3.27}$$

The last steps of the duality cascade can be recovered by taking this IR theory as starting point, and dualising up towards the UV. Some of these theories are shown in figure 13.

In the supergravity dual we would like to stick to large M, so that the probe approximation for the D7-branes is valid. From the field theory perspective, the system can be analysed even for relatively small M. The relevant dynamics is as follows. The dynamics is controlled by the $\mathrm{SU}(M+p_i)$ gauge groups, with other factors only weakly gauged, so they are regarded as global symmetries. Thus those gauge factors behave as decoupled $\mathrm{SU}(N_{c,i})$

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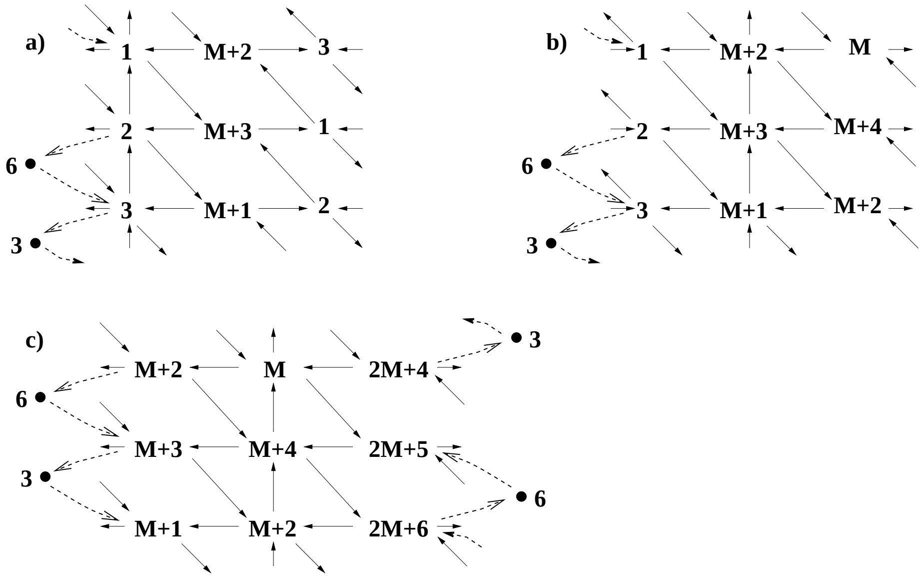

Figure 13: Quivers diagrams for the last steps in the duality cascade of the theory with flavours. In going from (b) to (c) we pushed the diagram one column to the left.

gauge theories with $N_{f,i}$ flavours, with $N_{f,i} < N_{c,i}$. Concretely the $\mathrm{SU}(M+3)$, $\mathrm{SU}(M+2)$, $\mathrm{SU}(M+1)$ nodes have 3, 4, and 5 flavours respectively. Each such node thus confines and develops an Affleck-Dine-Seiberg superpotential [33]. In terms of the corresponding mesons, which are defined as in (3.12), the total superpotential reads

$$\begin{aligned} W = {} & D_{i-1,i}A_{i,i-1} - C_{i,i+1}H_{i+1,i}\widetilde{H}_{i,i} + \widetilde{H}_{i,i}H_{i,i-1}\Phi_{i-1,i} - \Phi_{i,i+1}B_{i+1,i} + \\ & + M\left(\frac{\Lambda^{3M+6}_{\mathrm{SU}(M+3)}}{\det \mathcal{M}_{\mathrm{SU}(M+3)}}\right)^{\frac{1}{M}} + (M-3)\left(\frac{\Lambda^{3M+1}_{\mathrm{SU}(M+2)}}{\det \mathcal{M}_{\mathrm{SU}(M+2)}}\right)^{\frac{1}{M-3}} + \\ & + (M-3)\left(\frac{\Lambda^{3M-1}_{\mathrm{SU}(M+1)}}{\det \mathcal{M}_{\mathrm{SU}(M+1)}}\right)^{\frac{1}{M-3}} . \end{aligned}$$

As in the case without flavours, the non-perturbative superpotential forces the meson determinant to develop a vacuum expectation value. The analysis of this theory is somewhat analogous to the discussion in section (3.2.2), after (3.15). The pattern of symmetry breaking is (3.16) and leads to a $\mathrm{U}(3)\times\mathrm{U}(2)\times\mathrm{U}(1)$ gauge group. Computation of the light fields and their superpotential in the effective theory gives rise to a gauge theory associated with the quiver in figure 9. This is in complete agreement with the supergravity picture with the D3/D7-brane probe system.

The above construction realizes an almost conformal cascading gauge theory which, at an infrared scale, develops confinement leaving a set of light degrees of freedom with a structure strikingly close to the Standard Model. This is an explicit realization of Strassler's ideas, described in [5, 40], on the realization of the SM as the infrared of a duality cascade.

Hence the model should be regarded as a (supersymmetric) walking technicolor model. Moreover this theory is the holographic dual of the supergravity throat described above.

3.4 Further possibilities

Our aim has been developing general rules and techniques to construct warped throats containing singularities after complex deformation, and providing the relevant gauge theory dynamics matching the dual geometry. Clearly many other models, beyond those considered here, are possible. In this section we sketch some possibilities.

A simple modification is to consider other local configurations of D3-branes leading to semi-realistic models. In particular one may easily construct 3-family left-right symmetric models by simply making a different choice of Chan-Paton actions on the D3- and D7-branes of the model, following [21]. The discussion of the throat and its holographic dual cascade is completely analogous to the case we have studied. In this construction, the infrared gauge group is $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, and all quarks and Higgsses are composites, while leptons are fundamental (providing a tantalising suggestion of the quark-lepton Yukawa hierarchy). We leave a more detailed discussion as an exercise for the interested reader.

As an alternative kind of embedding, one may embed the SM on sets of D7-brane probes instead of D3-branes, but leading otherwise to similar spectra (e.g. as given in table 1, with 33 states in the table arising from 77 states). Although we do not describe this in detail, we would like to point that in the holographic description, 77 states like gauge bosons and the up quarks and up type Higgsses are bulk fields and thus fundamental, while 37 states like leptons and the down quarks and down type Higgsses sit at the bottom of the throat and are in fact composite. Thus this setup still may allow an explanation of the hierarchy, since e.g. the μ term is naturally suppressed. Moreover, bulk gauge bosons have been argued as a useful setup to address gauge coupling unification [41], and to suppress radiative corrections (by introducing a custodial SU(2) [42] e.g. by constructing left-right symmetric models, eventually broken by boundary conditions in the ultraviolet, namely the bulk of the Calabi-Yau compactification).

Another interesting possibility would be to embed the SM in a sector of anti-D-branes, which leads to non-supersymmetric models. In the supergravity picture this is straightforward, and the relevant techniques can be adapted from the local analysis in [21]. The holographic field theory description should involve a generalization of the non-supersymmetric baryonic branch studied in [43] for the conifold theory. One advantage of models with anti-D3-branes, as emphasised in [9] is that they naturally fall to the bottom of warped throats.

We leave these and other extensions for future research.

4. Multi-warp throats

In the previous section we have considered geometries which, after complex deformation, leave a non-trivial geometry which nevertheless does not admit further deformation. An important alternative class of throats is obtained if instead the left over singularity admits further deformations. This implies that in the presence of suitable fractional branes, the left over singularity develops a further throat, possibly ending in a further complex deformation. Such examples were described in [12], and shown to correspond to gauge theories with several scales of partial confinement. In this section we describe general techniques to construct geometrical setups of this kind, and briefly sketch their holographic field theory description.

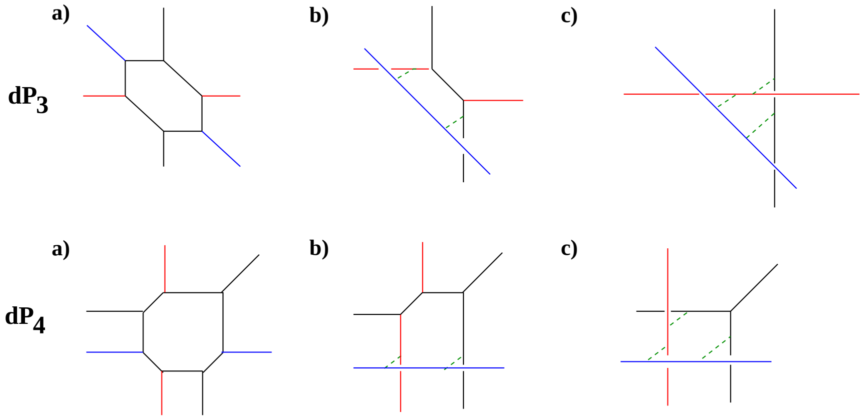

Figure 14: Two-parameter complex deformations for the complex cone over dP_3 and the complex cone over (a toric realization of) dP_4.

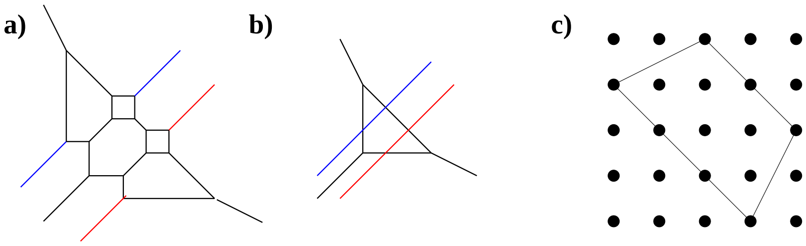

Figure 15: Web and toric diagrams for a geometry that, supplemented with fluxes, describes a two-warp throat ending on a $\mathbb{C}^3/\mathbb{Z}_3$ singularity. Figure (a) and (b) describe the generic web diagram, and the diagram at the origin of the deformation branch, respectively. Figure (c) shows the toric diagram, which indicates that the geometry is a $\mathbb{Z}_3$ quotient of the space $xy = z^2w^3$.

4.1 General construction

We are interested in warped throats with several radial regions of different warp factor. The complete geometry of such constructions are based on a warped version of a local Calabi-Yau geometry, which contains several 3-cycles of widely different size. Namely, the sizes of such 3-cycles are stabilized at hierarchically different values due to different 3-form fields. We will be more precise about this later on for a particular class of examples.

Hence the starting point we need is a singularity which admits several complex deformations. This is easily achieved using the techniques in section 2. Namely, we consider geometries associated to web diagrams which contain several sub-webs in equilibrium which can be removed from the picture. Reference [12] studied several examples, shown in figure 14, with two scales of deformation. It is straightforward to construct other geometries where there are more than two scales of deformation. Also, one can construct examples where there are several scales of deformation, leaving a left over geometry which contains a singularity admitting no further deformation. One such example is shown in figure 15. These geometries can therefore be used to construct throats with several regions of different warp factors, and with an infrared end given by a smooth or singular geometry. In the

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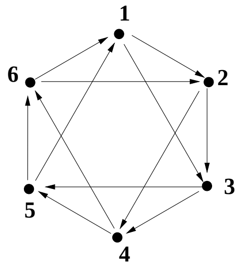

Figure 16: The quiver for D3-branes on the complex cone over dP_3.

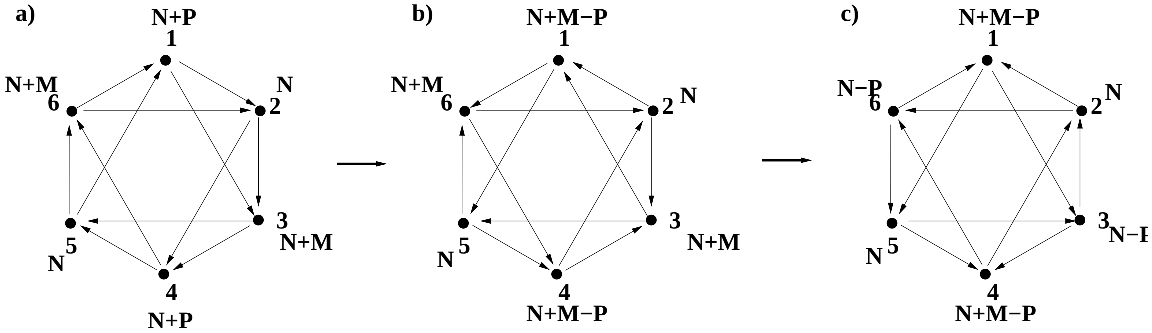

Figure 17: Two dualisations in the first RG cascade in dP_3.

latter case, the introduction of D-brane probes leads to chiral gauge theories at the bottom of a multi-warp throat, generalising the construction in previous section. Figure 15 provides a concrete example of one such geometry, which can be used to construct a two-warp throat ending on a $\mathbb{Z}_3$ orbifold singularity, at which one can localise the SM chiral gauge sector.[7]

Again, as soon as geometries become more involved it is non-trivial to construct the corresponding quiver gauge theories, which provide the holographic duals of these throats. Nevertheless, some examples of this kind have been achieved, and the relevant gauge theory dynamics is described in the next section.

4.2 Holographic description

In this section we review the simplest such example, studied in [12], which illustrates the general gauge theory dynamics underlying general multi-warp throats. The geometry is the complex cone over dP_3, which has a two-parameter set of complex deformations, shown in figure 14, in terms of the web diagram for the geometry (there is an alternative one-parameter branch of complex deformations [12] which will not interest us here).

[7]Moreover, the geometry corresponds to an orbifold of the geometry $xy = z^2w^3$, for which the dual quiver theory can be determined using tools in [23]. Hence, the field theory dual, the cascade flow and the infrared deformations for the multi-warp throat SM may be studied by a combination of the techniques in the previous and the coming sections. We leave this exercise for the interested reader.

The quiver theory for D3-branes at this singularity was described in [24], and corresponds to the diagram in figure 17a. The superpotential is

$$\begin{aligned} W \;=\; & X_{12}X_{23}X_{34}X_{45}X_{56}X_{61} + X_{13}X_{35}X_{51} + X_{24}X_{46}X_{62} - \\ & - X_{23}X_{35}X_{56}X_{62} - X_{13}X_{34}X_{46}X_{61} - X_{12}X_{24}X_{45}X_{51} \end{aligned} \tag{4.1}$$

in self-explanatory notation.

A basis of fractional branes is given by the rank vectors $(1,0,0,1,0,0)$, $(0,0,1,0,0,1)$ and $(1,0,1,0,1,0)$. The duality cascade associated to the two-warp throat, namely to the two-parameter complex deformation, corresponds to a choice of ranks

$$\vec{N} = N(1,1,1,1,1,1) + P(1,0,0,1,0,0) + M(0,0,1,0,0,1) \tag{4.2}$$

where we consider $N \gg P \gg M$. Given the $\mathbb{Z}_2$ symmetry of the quiver, a symmetric choice of gauge couplings in the ultraviolet is preserved under the RG flow, and the analysis of the cascade is simplified. The duality cascade proceeds as follows. The nodes with largest beta function are 1 and 4, so we dualise them simultaneously. The results are shown in figure 17a and 17b. Next the most strongly coupled nodes are 3, 6, so we dualise them simultaneously, as shown in figure 17b and 17c.

The quiver in figure 17c can be reordered into a quiver like the initial one, with similar fractional branes, but with the effective N reduced by an amount P. We can then continue dualising nodes 2, 5, then 1, 4, then 3, 6 etc, following the above pattern and generating a cascade which preserves the fractional branes but reduces the effective N.

The cascade proceeds until the effective N is not large compared with P. For simplicity, consider that the starting N is $N = (k+2)P - M$. Then after a suitable number of cascade steps, the ranks in the quiver are $(2P-M, P-M, P, 2P-M, P-M, P)$, for nodes 123456. In the next duality step the theory experiences partial confinement, and the quiver diagram of the theory collapses to a simpler one. This is the holographic dual of the first complex deformation ending the warped throat in the supergravity description.

In order to identify the remaining quiver theory and the corresponding geometry, we may consider the mesonic branch of this next-to-last step in the duality cascade, which describes the dynamics of D3-branes probes of the infrared theory. At this stage the $\mathrm{SU}(2P-M)$ factors have $2P-M$ flavours and develop a quantum deformation of their moduli space. Following the familiar procedure we introduce the mesons

$$\mathcal{M} = \begin{bmatrix} M_{63} & M_{62} \\ M_{53} & M_{52} \end{bmatrix} = \begin{bmatrix} X_{61}X_{13} & X_{61}X_{12} \\ X_{51}X_{13} & X_{51}X_{12} \end{bmatrix}; \qquad \mathcal{N} = \begin{bmatrix} N_{36} & N_{35} \\ N_{26} & N_{25} \end{bmatrix} = \begin{bmatrix} X_{34}X_{46} & X_{34}X_{45} \\ X_{24}X_{46} & X_{24}X_{45} \end{bmatrix}$$

and the baryons $\mathcal{B}$, $\widetilde{\mathcal{B}}$, $\mathcal{A}$, $\widetilde{\mathcal{A}}$. As usual we impose the quantum constraints via Lagrange multipliers X_1, X_2, and saturate them along the mesonic branch. The meson vevs break the symmetry to $\mathrm{SU}(P)\times\mathrm{SU}(P+M)$ where the first factor arises from nodes 2 and 5, and the second from 3 and 6. Restrict to the abelian case, the superpotential reads

$$\begin{aligned} W \;=\; & M_{62}X_{23}N_{35}X_{56} - X_{23}X_{35}X_{56}X_{62} - M_{63}N_{36} - M_{52}N_{25} + \\ & + M_{53}X_{35} + N_{26}X_{62} + M_{63}M_{52} - M_{53}M_{62} + N_{36}N_{25} - N_{26}N_{35}\,. \end{aligned} \tag{4.3}$$

Using the equations of motion for e.g. M_{53}, N_{26}, M_{63}, N_{25}, we are left with

$$W = X_{23}N_{35}X_{56}M_{62} - X_{23}M_{62}X_{56}N_{35}\,. \tag{4.4}$$

Going back to the non-abelian case, this is the gauge theory of D3-branes at a conifold singularity, showing that the left over geometry after the complex deformation is a conifold (as expected from the web diagram in the top part of figure 14. Since the conifold theory also contains fractional branes, the latter trigger a new duality cascade in which P is reduced in $2M$ units in each step [5]. This leads to a warped throat starting at the IR of the previous one, as in figure 1b.

4.3 Description a la RS

The general picture is thus a gauge theory with several stages of partial confinement, hence corresponding to throats with several warp regions. The horizon topology changes in crossing from one region to another, since it involves losing or gaining the finite-size 3-cycles in the transitions. A more detailed supergravity description in the bulk of each warp region is possible by simply taking the single-warp version of throats based on geometries with the corresponding 5d horizon [5, 10, 11]. Patching these together would lead to a description of the full throat, in the approximation of ignoring the finite extent of the transition regions. Unfortunately, even the set of known single warp solutions is relatively small.

In this section we provide an even more simplified description of the multi-warp throats, which may nevertheless be useful in applications. One ignores the internal 5d space and approximates each warp region by a slice of an AdS_5 of suitable radius, and glues them together at distances determined by the flux data of the real throat. This is in the spirit of approximating the KS throat by an AdS_5 geometry with an IR cutoff.

The description also reproduces RS constructions with several positive tension branes (corresponding to our transition regions), hence we may simply translate the results from the construction in e.g. [15] (modulo trivial notation changes). We consider several 'branes' located at positions $y_L < y_{L-1} < \cdots < y_1 < y_0$, $i = 0, \ldots, L$ in the fifth coordinate y (with larger y corresponding to further in the IR), which extends up to $y = y_0$. At each 'brane' we associate M_i, K_i units of RR and NSNS 3-form fluxes, so that the effective D3-brane number for $y_{i+1} < y < y_i$ is $N_i = M_iK_i$. Here and henceforth repeated indices are not summed. The metric is given by

$$ds^2 = e^{-2\sigma(y)}\, ds^2_{4d} + dy^2 \tag{4.5}$$

where $\sigma(y)$ is a continuous piecewise linear function of y, with slope $1/R_i$ in the interval $y_{i+1} < y < y_i$, with

$$R_i^4 = a_i g_s N_i \alpha'^2 \tag{4.6}$$

where a_i is a numerical coefficient depending on the 5d horizon in the real throat. Finally, in the real throat there is a linear running of the effective 5-form flux with y

$$\mathcal{K}(y) = N_i + \frac{b_i g_s M_i^2(-y + y_{i+1})}{R_i} \tag{4.7}$$

where b_i is a numerical coefficient depending on the properties of 3-cycles in the 5d horizon. Hence, the positions y_i are fixed by the flux data, by the condition that each warp region lasts until one runs out of 5-form flux. Hence, the boundaries of each warp region are related by

$$e^{-2y_i/R_i} = e^{-2y_{i+1}/R_i} \, e^{-\frac{K_i}{b_i g_s M_i}} \, . \tag{4.8}$$

Clearly the warp factor at the IR of the full throat is related to the UV one by a product of exponentials, as expected from the field theory analysis.

4.4 Applications

The model building applications of multi-warp throats are many, given their flexibility in generating several hierarchies. In this section we would like to simply point out some possibilities, whose details we leave for future research.

A possible application is the construction of brane inflation models [44, 45] where inflation takes place in the same throat in which the SM is located. This is not possible with traditional throats, since the warp factor required for inflation and for solving the Planck/electroweak hierarchy are widely different. This can be elegantly solved by the use of two-warp throats. Notice that, for models where the SM configuration requires D3-branes (as in our case), this scenario does not lead stable strings after reheating [46].

Another interesting possibility is the construction of inflationary models with several stages of inflation. This would manifest in the appearance of relatively abrupt changes in the slow-roll parameters, and therefore in the observables depending on them. This is nevertheless irrelevant unless the transition regions are crossed in the last 60 efoldings (or whatever number corresponding to the 'observable' stage of inflation).

Besides these cosmological applications, there may well be possibilities to exploit the multiple hierarchies in a multi-warp throat. Namely, using similar ideas it may be possible to construct throats with additional sectors localized in the transition regions. These could be exploited to generate new physics at interesting particle physics scales, like the intermediate scale or the GUT scale. We leave these interesting issues for further work.

5. Conclusions

In this paper we have developed techniques to construct warped throats with rich geometrical properties, as well as their holographic dual descriptions in terms of cascading RG flows ending on partial confinement.

We have proposed two interesting ideas, exploiting these richer geometries: on the one hand throats ending on singularities, with D-branes localized on them, and leading to chiral gauge sector at the IR end of the throat; on the other, multi-warp throats as in [12].

We have provided a quite explicit description of these systems. In particular we have described in detail the holographic dual of a throat ending on a system of D3/D7-branes at a $\mathbb{C}^3/\mathbb{Z}_3$ singularity, realizing a 3-family SM like gauge sector. Independently of the string theory setup, the field theory provides the construction of an almost conformal UV gauge theory which partially confines at low energies, and leaves the SM as its light degrees of freedom. This is an explicit realization of the SM as a composite model, in the IR of a duality cascade (along ideas in [40]).

These results are remarkable. Nevertheless, many open questions remain. It would be desirable to find mechanisms to break supersymmetry while retaining control of the throat geometry. It would also be interesting to adapt these ideas to model building with more flexible brane configurations, like intersecting or magnetised D-branes. In this respect the local models in [47] could be a good starting point. Finally, it would be extremely interesting to sharpen the understanding of throats that we have, namely by finding additional explicit metrics for horizon manifolds, or the details of the backreaction of D7-branes in models containing them.

We expect that, based on this work and on a more profound understanding of the gauge dynamics underlying warped throats, these and other open questions can be addressed, leading to improved model building potential for these constructions.

Acknowledgments

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A. The SPP and its cascade

A.1 The SPP field theory and moduli space

The gauge theory on D3-branes at a suspended pinch point (SPP) singularity [22, 23] has gauge group $\mathrm{U}(N_1) \times \mathrm{U}(N_2) \times \mathrm{U}(N_3)$ and matter content:

	$\mathrm{U}(N_1)$	$\mathrm{U}(N_2)$	$\mathrm{U}(N_3)$
F	$\square$	$\overline{\square}$	1
$\widetilde{F}$	$\overline{\square}$	$\square$	1
G	1	$\square$	$\overline{\square}$
$\widetilde{G}$	1	$\overline{\square}$	$\square$
H	$\overline{\square}$	1	$\square$
$\widetilde{H}$	$\square$	1	$\overline{\square}$
Φ	Adj.	1	1

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The superpotential is

$$W = \mathrm{tr}\left(\widetilde{F}FG\widetilde{G} - \widetilde{G}GH\widetilde{H} + \widetilde{H}H\Phi - \Phi F\widetilde{F}\right) \tag{A.1}$$

where bi-fundamental fields are regarded as matrices. The quiver diagram is shown in figure 18.

When $N_1 = N_2 = N_3 \equiv N$, the generic mesonic branch describes the motion of N D3-branes on the SPP geometry. Thus, for $N = 1$ the moduli space is given by the SPP geometry itself. This can be shown using the techniques in [20], as done explicitly in [22, 36].

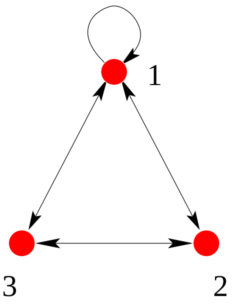

Figure 18: Quiver diagram for SPP.

The possible vevs for the above fields are constrained by the F-term equations from the superpotential. These can be satisfied automatically if we express the above fields in terms of a set of new variables, p_i, with $i = 1, \ldots, 6$, as follows

$$\begin{aligned} F &= p_1 p_5\,, & \widetilde{F} &= p_2 p_3\,, & G &= p_4\,, & \widetilde{G} &= p_6 \\ H &= p_3 p_5\,, & \widetilde{H} &= p_1 p_2\,, & \Phi &= p_4 p_6\,. \end{aligned} \tag{A.2}$$

Thus, relations like

$$\frac{\partial W}{\partial \widetilde{H}} = \widetilde{G}GH - H\Phi = 0\,, \quad \text{etc.} \tag{A.3}$$

are automatically satisfied when using (A.2).

The moduli space is described by gauge invariant operators, modulo F-term relations. Namely, combinations of the fields modulo relations obtained when expressed in terms of (A.2). The corresponding monomials can be chosen e.g. as

$$\begin{aligned} x &= \widetilde{H}\widetilde{G}\widetilde{F} = p_1 p_2^2 p_3 p_6 \\ y &= FGH = p_1 p_3 p_4 p_5^2 \\ z &= \Phi = p_4 p_6 \\ w &= F\widetilde{F} = p_1 p_2 p_3 p_5 \end{aligned} \tag{A.4}$$

where of course the F-term relation also allow to say e.g. $z = G\widetilde{G}$, etc.

The monomial satisfy the relation

$$xy = zw^2 \tag{A.5}$$

hence parametrise a moduli space which corresponds to the SPP singularity.

In the more technical but powerful language of toric geometry, the moduli space is constructed as the symplectic quotient associated to the toric data [22, 36]:

	p_1	p_2	p_3	p_4	p_5	p_6
Q_F	1	-1	1	0	-1	0
Q_2	-1	1	0	1	0	-1
Q_3	-1	0	0	-1	1	1

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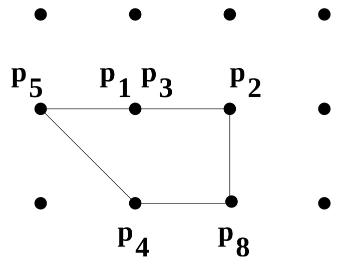

Figure 19: Toric diagram for the SPP geometry.

Namely the moduli space is parametrised by vevs for the p_i's modulo the gauge equivalence by the $\mathrm{U}(1)^3$ symmetry with the above charge assignment. The Q_F charges are constructed so that the fields of the SPP theory are neutral; hence Q_F-gauge invariant vevs for the p_i's can be associated to vevs for the SPP fields, satisfying the F-term constraints. The Q_2, Q_3 charges impose the D-term constraints of the SPP theory at the level of the p_i's. Hence the $\mathrm{U}(1)^3$-gauge invariant vevs for p_i's correspond to F- and D-flat vevs for the SPP theory.

The toric diagram for the resulting geometry is easily found from the toric data above. Namely, the points in the toric diagram have coordinates given by the columns in the cokernel matrix

$$T = \begin{pmatrix} 0 & 1 & 0 & 0 & -1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}. \tag{A.6}$$

All points lie on a plane, as should be for a Calabi-Yau geometry. The points on the plane are shown in figure 19a

A.2 The SPP duality cascade

The existence of a duality cascade for the SPP field theory in the presence of fractional branes has been established and discussed in [12].

The ranks of the gauge factors are arbitrary, hence there are two independent fractional branes, which can be taken to be $(0,1,0)$ and $(0,0,1)$. Let us consider the starting point given by the ranks

$$\vec{N} = N(1,1,1) + M(0,1,0)\,. \tag{A.7}$$

By following the pattern of dualising the most strongly coupled node at each step, we are led to a cascade that repeats the following sequence of dualisations $(2,1,3,2,1,3)$. The quiver theories at each step of this sequence are shown in figure 20. We have indicated in blue the node that gets dualised at each step. After six dualisations, the quiver comes back to itself, with $N \to N-3M$ and M constant. This pattern of dualisations is consistent with a RG flow, as discussed in [12].

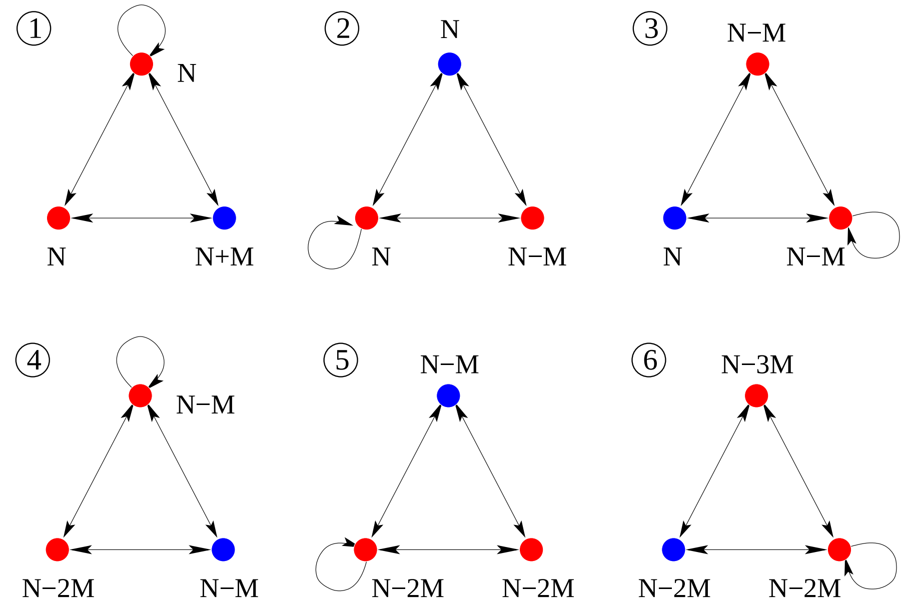

Figure 20: Sequence of quivers in one period of the SPP cascade. We have indicated in blue the dualised node at each step.

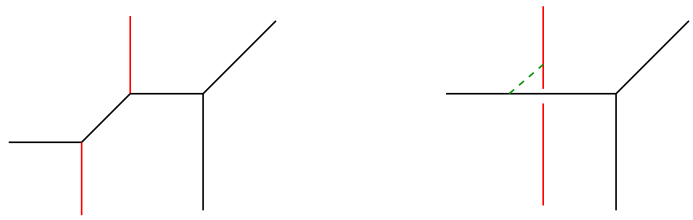

Figure 21: Web diagram for the SPP and its deformation to a smooth geometry.

A.3 The infrared deformation

As usual, the cascade proceeds until the effective number of D3-branes is comparable with M. After this we expect the gauge theory strong dynamics to take over and induce a geometric transition. Indeed, the SPP singularity admits a complex deformation. This is manifest using the web diagram for the SPP geometry, figure 21*a*, which contains a sub-web in equilibrium (a line) which may be removed from the picture, as shown in figure 21*b*. In the following we describe how this arises in the field theory, following [12].

In order to study the infrared end of the cascade, we study the gauge theory describing M D3-branes probing it. This corresponds to the quiver theory with rank vector

$$\vec{N} = M(1,1,1) + M(0,1,0)\,. \tag{A.8}$$

In this case, we only need to consider mesons for node 2, given by

$$\mathcal{M} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} FG & F\widetilde{F} \\ \widetilde{G}G & \widetilde{G}\widetilde{F} \end{bmatrix} \tag{A.9}$$

and the baryons $\mathcal{B}$, $\widetilde{\mathcal{B}}$. The quantum constraint in the superpotential reads

$$\det \mathcal{M} - \mathcal{B}\widetilde{\mathcal{B}} = \Lambda^{4M} \,. \tag{A.10}$$

To derive the dynamics of the M probes, we choose the mesonic branch

$$X = \Lambda^{4-4M} \,; \qquad \mathcal{B} = \widetilde{\mathcal{B}} = 0 \,; \qquad \det \mathcal{M} = \Lambda^{4M} \,. \tag{A.11}$$

Restricting to the abelian case, the superpotential reads

$$W = \operatorname{tr}\left(-CH\widetilde{H} + \widetilde{H}H\Phi - \Phi B + BC\right). \tag{A.12}$$

The equation of motion for B requires $\Phi = C$, so we get

$$W = -CH\widetilde{H} + \widetilde{H}HC. \tag{A.13}$$

The gauge group (in the non-abelian case) is $\mathrm{U}(M)$ (due to the breaking by meson vevs $M \propto \mathbb{1}$). All three fields transform in the adjoint representation (a singlet in the abelian case). The above theory clearly describes the field content and superpotential of $\mathcal{N} = 4$ SYM, i.e. the theory describing the smooth geometry left over after the deformation.

In addition, there remain some additional light fields, namely A, D, B, subject to the constraint

$$AD - BC = \Lambda^4 \,. \tag{A.14}$$

The dynamics is that of probe D3-branes in the geometry corresponding to the deformation of the SPP to flat space. This matches nicely the geometric expectation, from the web diagrams in figure 21, from which we see that the result of the deformation is a smooth geometry.

As we have mentioned, for $N = KM$ the field theory is dual to a pure supergravity throat with no extra branes (this is achieved by taking the baryonic branch in the above analysis of the $\mathrm{SU}(M) \times \mathrm{SU}(2M) \times \mathrm{SU}(M)$ theory). In the main applications in this paper, we are interested in throats with a number $p < M$ of D3-brane probes at its tip. Following [5], the holographic duals of such configurations correspond to the same quiver field theory but with $N = KM + p$. In this situation, the next to last step of the cascade corresponds to an $\mathrm{SU}(p) \times \mathrm{SU}(M + p) \times \mathrm{SU}(p)$ theory. Considering the $\mathrm{SU}(p)$ theories as global symmetries, we have a $\mathrm{SU}(M + p)$ gauge theory with $2p$ flavours. Since it has less flavours than colors, the theory confines, leaving the mesons (A.9) as light degrees of freedom, with a non-perturbative Affleck-Dine-Seiberg superpotential [33]

$$W_{ADS} = (N_f - N_c)\left(\frac{\Lambda^{3N_c - N_f}}{\det \mathcal{M}}\right)^{\frac{1}{N_c - N_f}} \,. \tag{A.15}$$

Following [5] the superpotential forces $\det \mathcal{M}$ to acquire a non-zero vev, which breaks the symmetry down to $\mathrm{SU}(p)$. The left over fields are three chiral multiplets in the adjoint, described by an $\mathcal{N} = 4$ SYM theory. This is the gauge theory on the p D3-branes sitting at

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the bottom of the throat. This analysis is particularly simple for $p = 1$ (and in particular has appeared in [28]). The complete superpotential reads

$$W = AD - CH\widetilde{H} + \widetilde{H}H\Phi - \Phi B + (M-1)\left(\frac{\Lambda^{3M+1}}{AD-BC}\right)^{\frac{1}{M-1}} . \qquad \text{(A.16)}$$

The equation of motion for e.g. A shows that $\det \mathcal{M} = AD - BC = \Lambda^{\frac{3M+1}{M}}$. This triggers the promised breaking of the symmetry, in which A and D are eaten up. Setting $\Lambda = 1$ for simplicity in what follows, the equation of motion for B imposes $\Phi = C$. Replacing these relations in the above superpotential, the left over adjoint fields H, $\widetilde{H}$, C have $\mathcal{N} = 1$ superpotential interactions, as in (A.13). Notice that although in the abelian case it vanishes identically, it is crucial for higher p.

As mentioned in [12], the relation between the field theory and the geometrical description of the deformation can be obtained using the construction of the moduli space of the SPP in section A.1. The gauge invariant operators (A.4), expressed in terms of the mesons, read

$$x = \widetilde{H}D\,, \qquad y = AH\,, \qquad z = \Phi = C\,, \qquad w = B = \widetilde{H}H \qquad \text{(A.17)}$$

where we have used F-term relations to express some operator in several equivalent ways. The monomials satisfy $xy - zw^2 = 0$ at the classical level, namely

$$\widetilde{H}H(AD - BC) = 0\,. \qquad \text{(A.18)}$$

On the other hand, at the non-perturbative level we have $AD - BC = \epsilon$. Hence, the moduli space corresponds to

$$\widetilde{H}H(AD - BC) = \epsilon\widetilde{H}H\,. \qquad \text{(A.19)}$$

Namely

$$xy - zw^2 = \epsilon w\,. \qquad \text{(A.20)}$$

This is a complex deformation of the SPP to a smooth space.

A.4 From the parent to the $\mathbb{Z}_3$ quotient theory

The quotient on the geometry. As discussed in the main text, the toric data of the geometry of interest indicates that it is a $\mathbb{Z}_3$ quotient of the SPP. Here we would like to be more explicit about this point.

Given a set of toric data for a geometry, namely the charges of a set of p_i's under a set of U(1) symmetries, there is a systematic way to construct the toric data of a $\mathbb{Z}_N$ quotient of it. The generator θ of $\mathbb{Z}_N$ has an action on the p_i's given by a phase rotation $e^{2\pi i a_i/N}$ (which is defined modulo the U(1) gauge invariances), with $\sum_i a_i = N$. The quotient is described by adding a new variable P, uncharged under the old gauge symmetries, and a new U(1) gauge symmetry, under which the p_i have charges a_i/N and P has charge -1. The idea is that one can use the new U(1) symmetry and its D-term to eliminate P. This however still leaves a $\mathbb{Z}_N$ subgroup of the U(1) (phase rotations by 2π) acting trivially on

P but non-trivially on the p_i's. Modding by this discrete gauge symmetry thus implements the orbifold quotient, with the desired action.

We can apply this to show that the quotient of the SPP by the $\mathbb{Z}_3$ action

$$p_5 \to \alpha^2 p_5 \,; \qquad p_6 \to \alpha p_6 \tag{A.21}$$

(modulo gauge equivalences) leads to the toric data of the geometry used in the main text. Following the above procedure we add a new field and gauge symmetry to (A.1):

$$\begin{array}{cccccccc} & p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & P \\ Q_F & 1 & -1 & 1 & 0 & -1 & 0 & 0 \\ Q_2 & -1 & 1 & 0 & 1 & 0 & -1 & 0 \\ Q_3 & -1 & 0 & 0 & -1 & 1 & 1 & 0 \\ Q_{\mathbb{Z}_3} & 0 & 0 & 0 & 0 & 2/3 & 1/3 & -1 \end{array}$$

The toric diagram is obtained by computing the cokernel of the charge matrix, namely

$$\widetilde{T} = \begin{pmatrix} 0 & 0 & 0 & 3 & 0 & 3 & 1 \\ 1 & 2 & 1 & 2 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \tag{A.22}$$

which reproduces figure 6 up to a linear transformation.

The action on the complex variables x, y, z, w can be obtained by use of (A.4), and reproduces the one described in the main text (3.3). Also the action on the fields of the SPP theory (3.6) used in the main text is obtained by using (A.2).

The quotient field theory. Most results of the main text concerning the quotient of the SPP, and the corresponding quiver gauge field theory, can be obtained by considering similar effects in the parent SPP field theory, and carrying out the $\mathbb{Z}_3$ quotient.[8] We describe this in the present section.

The construction of the field theory of the quotient of the SPP as a quotient of the field theory of the SPP has been described in the main text, section (3.2.2), and will not be repeated here.

The duality cascade of the quotient theory is simply the quotient of the cascade for the SPP theory, studied above. In order to show this it is enough to show that Seiberg duality and orbifolding commute. Sketchily, one can start with the SPP theory and perform a Seiberg duality on a node. Since the $\mathbb{Z}_3$ is a global symmetry, it exists in both theories, and its action of the original and dual fields is related. Carrying out the corresponding $\mathbb{Z}_3$ quotient, one is led to two theories for the quotient of the SPP theory. Both quotient theories are related by (simultaneous) Seiberg duality on the three nodes corresponding to the node dualised in the parent theory. We carry out the exercise in what follows.

[8]This is true for any final configuration in the quotient of the SPP which respects the $\mathbb{Z}_3$ quantum symmetry. This is not the case e.g. in section 3.3, where nodes in the quotient arising from a single node in the SPP are given different rank. Namely, the model contains fractional D3- and D7-branes with respect to the $\mathbb{Z}_3$ quotient.

Consider the field theory for the SPP (A.1). For illustration, consider the case with ranks $(N, N+M, N)$, which corresponds to the first step in the duality cascade. Carrying out the dualization of the node $\mathrm{SU}(N+M)$, we obtain the field theory:

	$\mathrm{SU}(N)$	$\mathrm{SU}(N-M)$	$\mathrm{SU}(N)$
F_d	$\overline{\square}$	$\square$	1
$\widetilde{F}_d$	$\square$	$\overline{\square}$	1
G_d	1	$\overline{\square}$	$\square$
$\widetilde{G}_d$	1	$\square$	$\overline{\square}$
H	$\overline{\square}$	1	$\square$
$\widetilde{H}$	$\square$	1	$\overline{\square}$
Φ'	1	1	Adj

In the dualization process, we have replaced quarks $\widetilde{F}$, G and antiquarks F, $\widetilde{G}$ of $\mathrm{SU}(N+M)$ by dual quarks F_d, $\widetilde{G}_d$ of $\mathrm{SU}(N-M)$, and we have introduced the mesons. However the mesons FG, $\widetilde{G}\widetilde{F}$ get massive due to the superpotential, and the meson $F\widetilde{F}$ becomes massive with the original adjoint Φ. The only remaining light meson is $\Phi' = \widetilde{G}G$.

The $\mathbb{Z}_3$ action (3.6) on the initial theory is a global symmetry, hence a corresponding action is induced on the dual theory. The action is obtained by noticing that quarks and dual (anti)quarks transform oppositely under global symmetries, and that Φ' is a meson in terms of the original quarks. The action of the dual is thus

$$\begin{aligned} F_d &\to \alpha F_d\,; & \widetilde{F}_d &\to \widetilde{F}_d \\ G_d &\to G_d\,; & \widetilde{G}_d &\to \alpha^2 \widetilde{G}_d \\ H &\to \alpha^2 H\,; & \widetilde{H} &\to \widetilde{H} \\ \Phi' &\to \alpha \Phi'\,. \end{aligned} \tag{A.23}$$

It is now possible to carry out the $\mathbb{Z}_3$ quotient of the dual theory by this action. The resulting theory is exactly given by figure 8b, namely the theory obtained after carrying out simultaneous Seiberg duality of the three $\mathrm{SU}(N+M)$ nodes of the $\mathbb{Z}_3$ quotient of the initial SPP theory. This completes our argument that orbifolding and Seiberg duality commute. Iteration of the argument leads to inheritance of the SPP cascade to the quotient theory.

Finally, it is straightforward to compare the computations of the geometric deformation from the field theory, in sections (3.2.2) and (A.3), and conclude that the former is directly inherited from the latter by a $\mathbb{Z}_3$ quotient. We leave the details to the interested reader.

References

[1] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[2] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[3] K. Dasgupta, G. Rajesh and S. Sethi, *M-theory, orientifolds and G-flux*, *JHEP* **08** (1999) 023 [hep-th/9908088].

[4] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[5] I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and χSB-resolution of naked singularities*, *JHEP* **08** (2000) 052 [hep-th/0007191].

[6] H.L. Verlinde, *Holography and compactification*, *Nucl. Phys.* **B 580** (2000) 264 [hep-th/9906182].

[7] I.R. Klebanov and E. Witten, *Superconformal field theory on threebranes at a Calabi-Yau singularity*, *Nucl. Phys.* **B 536** (1998) 199 [hep-th/9807080].

[8] N. Seiberg, *Electric - magnetic duality in supersymmetric nonabelian gauge theories*, *Nucl. Phys.* **B 435** (1995) 129 [hep-th/9411149].

[9] J.F.G. Cascales, M.P. Garcia del Moral, F. Quevedo and A.M. Uranga, *Realistic D-brane models on warped throats: fluxes, hierarchies and moduli stabilization*, *JHEP* **02** (2004) 031 [hep-th/0312051].

[10] S. Franco, Y.-H. He, C. Herzog and J. Walcher, *Chaotic duality in string theory*, *Phys. Rev.* **D 70** (2004) 046006 [hep-th/0402120].

[11] C.P. Herzog, Q.J. Ejaz and I.R. Klebanov, *Cascading RG flows from new sasaki-einstein manifolds*, *JHEP* **02** (2005) 009 [hep-th/0412193].

[12] S. Franco, A. Hanany and A.M. Uranga, *Multi-flux warped throats and cascading gauge theories*, *JHEP* **09** (2005) 028 [hep-th/0502113].

[13] C. Vafa, *Superstrings and topological strings at large-N*, *J. Math. Phys.* **42** (2001) 2798 [hep-th/0008142];
F. Cachazo, K.A. Intriligator and C. Vafa, *A large-N duality via a geometric transition*, *Nucl. Phys.* **B 603** (2001) 3 [hep-th/0103067];
F. Cachazo, S. Katz and C. Vafa, *Geometric transitions and $N = 1$ quiver theories*, hep-th/0108120;
F. Cachazo, B. Fiol, K.A. Intriligator, S. Katz and C. Vafa, *A geometric unification of dualities*, *Nucl. Phys.* **B 628** (2002) 3 [hep-th/0110028].

[14] I. Oda, *Mass hierarchy from many domain walls*, *Phys. Lett.* **B 480** (2000) 305 [hep-th/9908104]; *Mass hierarchy and trapping of gravity*, *Phys. Lett.* **B 472** (2000) 59 [hep-th/9909048].

[15] H. Hatanaka, M. Sakamoto, M. Tachibana and K. Takenaga, *Many-brane extension of the Randall-Sundrum solution*, *Prog. Theor. Phys.* **102** (1999) 1213 [hep-th/9909076].

[16] M. Aganagic and C. Vafa, *G_2 manifolds, mirror symmetry and geometric engineering*, hep-th/0110171.

[17] O. Aharony and A. Hanany, *Branes, superpotentials and superconformal fixed points*, *Nucl. Phys.* **B 504** (1997) 239 [hep-th/9704170];
O. Aharony, A. Hanany and B. Kol, *Webs of (p,q) 5-branes, five dimensional field theories and grid diagrams*, *JHEP* **01** (1998) 002 [hep-th/9710116].

[18] N.C. Leung and C. Vafa, *Branes and toric geometry*, *Adv. Theor. Math. Phys.* **2** (1998) 91 [hep-th/9711013].

[19] M.R. Douglas and G.W. Moore, *D-branes, quivers and ALE instantons*, hep-th/9603167.

[20] M.R. Douglas, B.R. Greene and D.R. Morrison, *Orbifold resolution by D-branes*, *Nucl. Phys.* **B 506** (1997) 84 [hep-th/9704151].

[21] G. Aldazabal, L.E. Ibáñez, F. Quevedo and A.M. Uranga, *D-branes at singularities: a bottom-up approach to the string embedding of the standard model*, *JHEP* **08** (2000) 002 [hep-th/0005067].

[22] D.R. Morrison and M.R. Plesser, *Non-spherical horizons, I*, *Adv. Theor. Math. Phys.* **3** (1999) 1 [hep-th/9810201].

[23] A.M. Uranga, *Brane configurations for branes at conifolds*, *JHEP* **01** (1999) 022 [hep-th/9811004].

[24] B. Feng, A. Hanany and Y.-H. He, *D-brane gauge theories from toric singularities and toric duality*, *Nucl. Phys.* **B 595** (2001) 165 [hep-th/0003085]; *Phase structure of D-brane gauge theories and toric duality*, *JHEP* **08** (2001) 040 [hep-th/0104259];
A. Hanany and A. Iqbal, *Quiver theories from d6-branes via mirror symmetry*, *JHEP* **04** (2002) 009 [hep-th/0108137].

[25] J.P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, *Sasaki-Einstein metrics on S^2xS^3*, *Adv. Theor. Math. Phys.* **8** (2004) 711 [hep-th/0403002];
D. Martelli and J. Sparks, *Toric geometry, sasaki-einstein manifolds and a new infinite class of AdS/CFT duals*, hep-th/0411238;
M. Bertolini, F. Bigazzi and A.L. Cotrone, *New checks and subtleties for AdS/CFT and a-maximization*, *JHEP* **12** (2004) 024 [hep-th/0411249];
S. Benvenuti, S. Franco, A. Hanany, D. Martelli and J. Sparks, *An infinite family of superconformal quiver gauge theories with Sasaki-Einstein duals*, *JHEP* **06** (2005) 064 [hep-th/0411264];
S. Benvenuti, A. Hanany and P. Kazakopoulos, *The toric phases of the $Y^{p,q}$ quivers*, *JHEP* **07** (2005) 021 [hep-th/0412279].

[26] Private communication from S. Franco and A. Hanany.

[27] B. Feng, S. Franco, A. Hanany and Y.-H. He, *Unhiggsing the del Pezzo*, *JHEP* **08** (2003) 058 [hep-th/0209228].

[28] H. Ita, H. Nieder and Y. Oz, *On type-II strings in two dimensions*, *JHEP* **06** (2005) 055 [hep-th/0502187].

[29] A. Sagnotti, *A note on the Green-Schwarz mechanism in open string theories*, *Phys. Lett.* **B 294** (1992) 196 [hep-th/9210127].

[30] L.E. Ibáñez, R. Rabadán and A.M. Uranga, *Anomalous* U(1)*'s in type-I and type-IIB $D = 4$, $N = 1$ string vacua*, *Nucl. Phys.* **B 542** (1999) 112 [hep-th/9808139].

[31] B. Feng, A. Hanany, Y.-H. He and A.M. Uranga, *Toric duality as Seiberg duality and brane diamonds*, *JHEP* **12** (2001) 035 [hep-th/0109063].

[32] C.E. Beasley and M.R. Plesser, *Toric duality is Seiberg duality*, *JHEP* **12** (2001) 001 [hep-th/0109053].

[33] I. Affleck, M. Dine and N. Seiberg, *Dynamical supersymmetry breaking in supersymmetric QCD*, *Nucl. Phys.* **B 241** (1984) 493.

[34] A. Karch and E. Katz, *Adding flavor to AdS/CFT*, *JHEP* **06** (2002) 043 [hep-th/0205236].

JHEP11(2005)047

[35] J.F.G. Cascales and A.M. Uranga, *Branes on generalized calibrated submanifolds*, *JHEP* **11** (2004) 083 [hep-th/0407132].

[36] J. Park, R. Rabadán and A.M. Uranga, *Orientifolding the conifold*, *Nucl. Phys.* **B 570** (2000) 38 [hep-th/9907086].

[37] A.M. Uranga, *From quiver diagrams to particle physics*, hep-th/0007173.

[38] R.G. Leigh and M. Rozali, *Brane boxes, anomalies, bending and tadpoles*, *Phys. Rev.* **D 59** (1999) 026004 [hep-th/9807082];
G. Aldazabal, D. Badagnani, L.E. Ibáñez and A.M. Uranga, *Tadpole versus anomaly cancellation in $D = 4, 6$ compact IIB orientifolds*, *JHEP* **06** (1999) 031 [hep-th/9904071];
M. Bianchi and J.F. Morales, *Anomalies and tadpoles*, *JHEP* **03** (2000) 030 [hep-th/0002149].

[39] P. Ouyang, *Holomorphic D7-branes and flavored $N = 1$ gauge theories*, *Nucl. Phys.* **B 699** (2004) 207 [hep-th/0311084].

[40] M.J. Strassler, *Duality in supersymmetric field theory: general conceptual background and an application to real particle physics*, in *Nagoya 1996, Perspectives of strong coupling gauge theories* 237.

[41] L. Randall and M.D. Schwartz, *Quantum field theory and unification in AdS5*, *JHEP* **11** (2001) 003 [hep-th/0108114]; *Unification and the hierarchy from AdS5*, *Phys. Rev. Lett.* **88** (2002) 081801 [hep-th/0108115].

[42] K. Agashe, A. Delgado, M.J. May and R. Sundrum, *RS1, custodial isospin and precision tests*, *JHEP* **08** (2003) 050 [hep-ph/0308036].

[43] S. Kachru, J. Pearson and H.L. Verlinde, *Brane/flux annihilation and the string dual of a non- supersymmetric field theory*, *JHEP* **06** (2002) 021 [hep-th/0112197].

[44] G.R. Dvali, Q. Shafi and S. Solganik, *D-brane inflation*, hep-th/0105203;
C.P. Burgess et al., *The inflationary brane-antibrane universe*, *JHEP* **07** (2001) 047 [hep-th/0105204].

[45] S. Kachru et al., *Towards inflation in string theory*, *JCAP* **10** (2003) 013 [hep-th/0308055].

[46] E.J. Copeland, R.C. Myers and J. Polchinski, *Cosmic F- and D-strings*, *JHEP* **06** (2004) 013 [hep-th/0312067].

[47] A.M. Uranga, *Local models for intersecting brane worlds*, *JHEP* **12** (2002) 058 [hep-th/0208014].

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